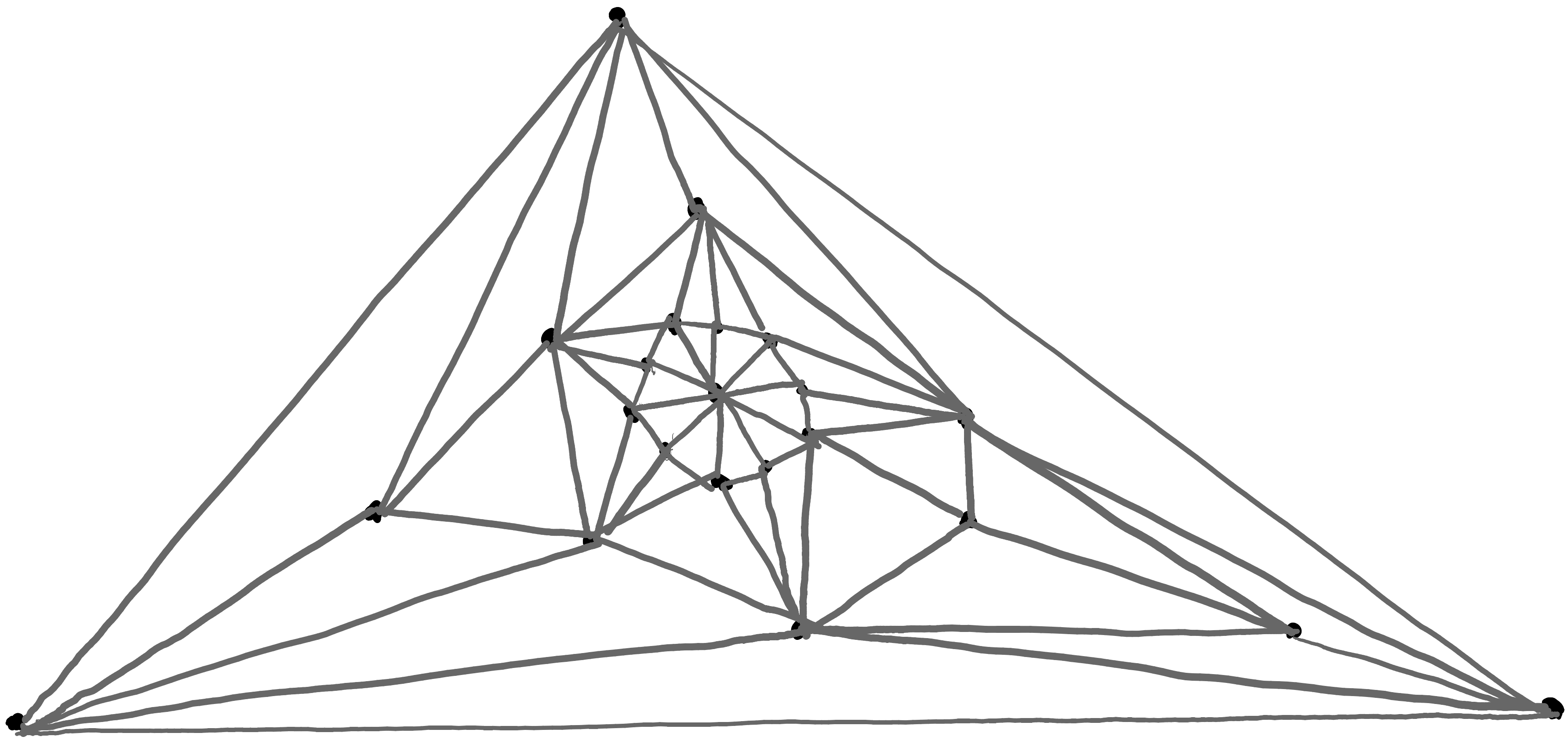
Kirkpatrick's Method

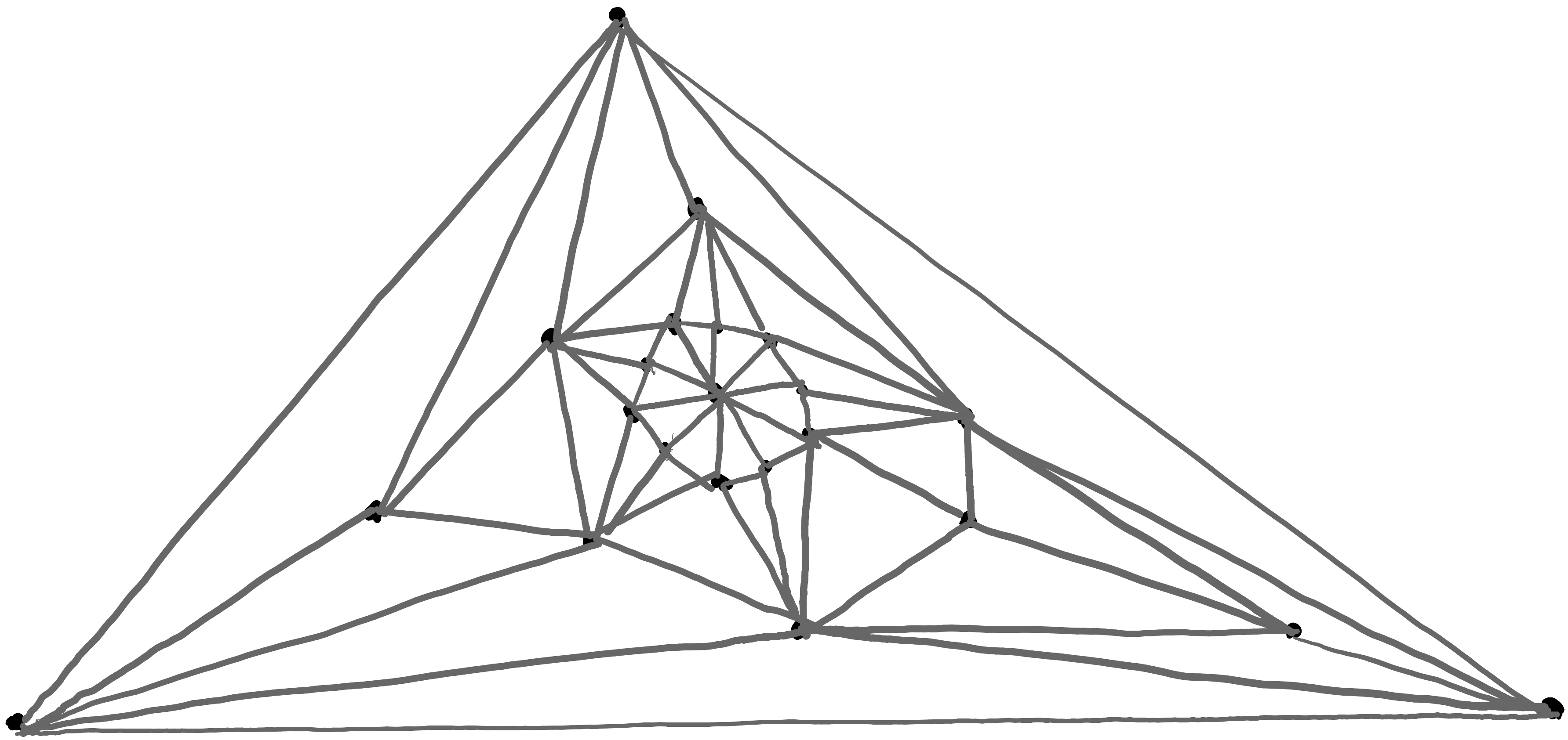
SIAM J. Comput 12(1) 1983

John Iacono

The Problem



Input: A triangulation



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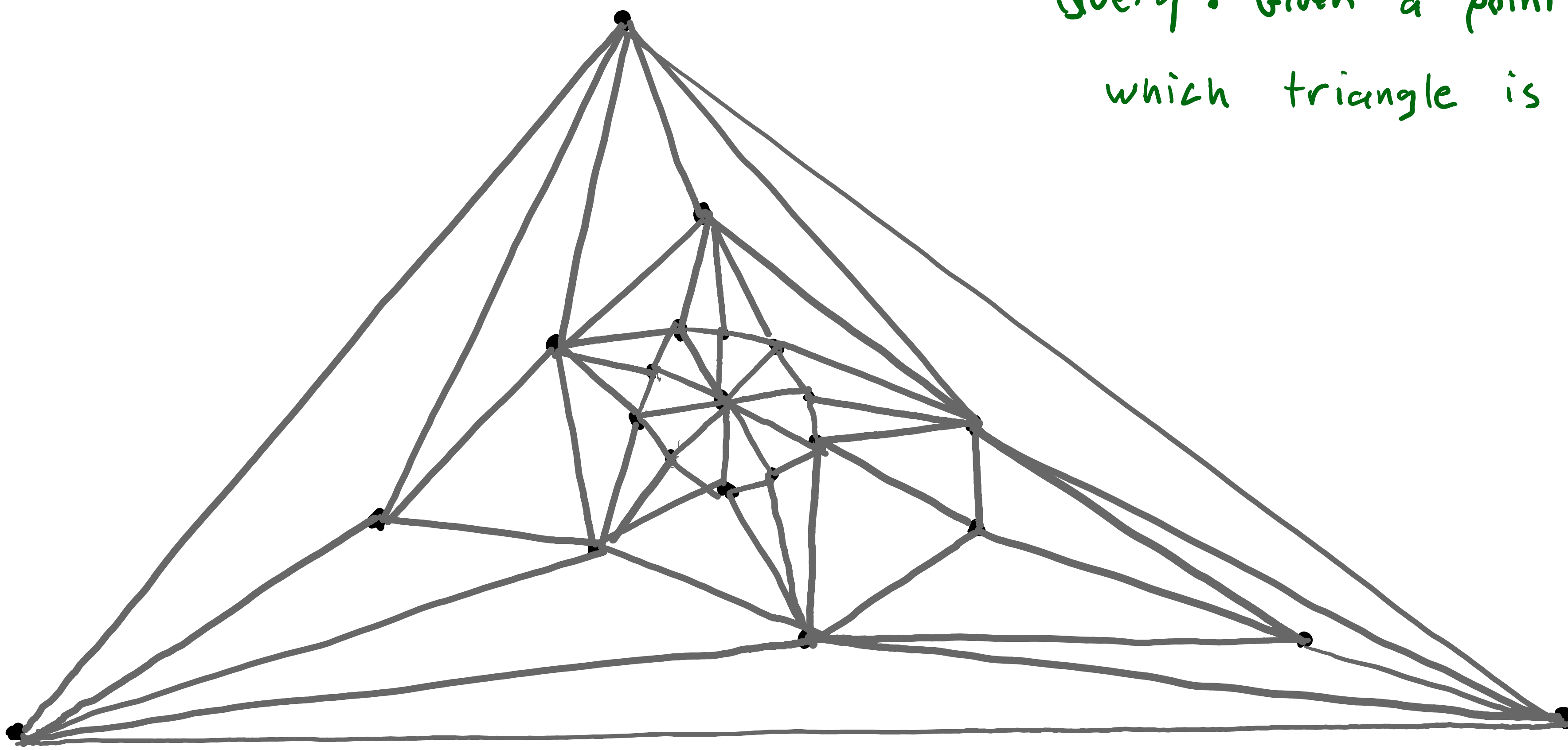

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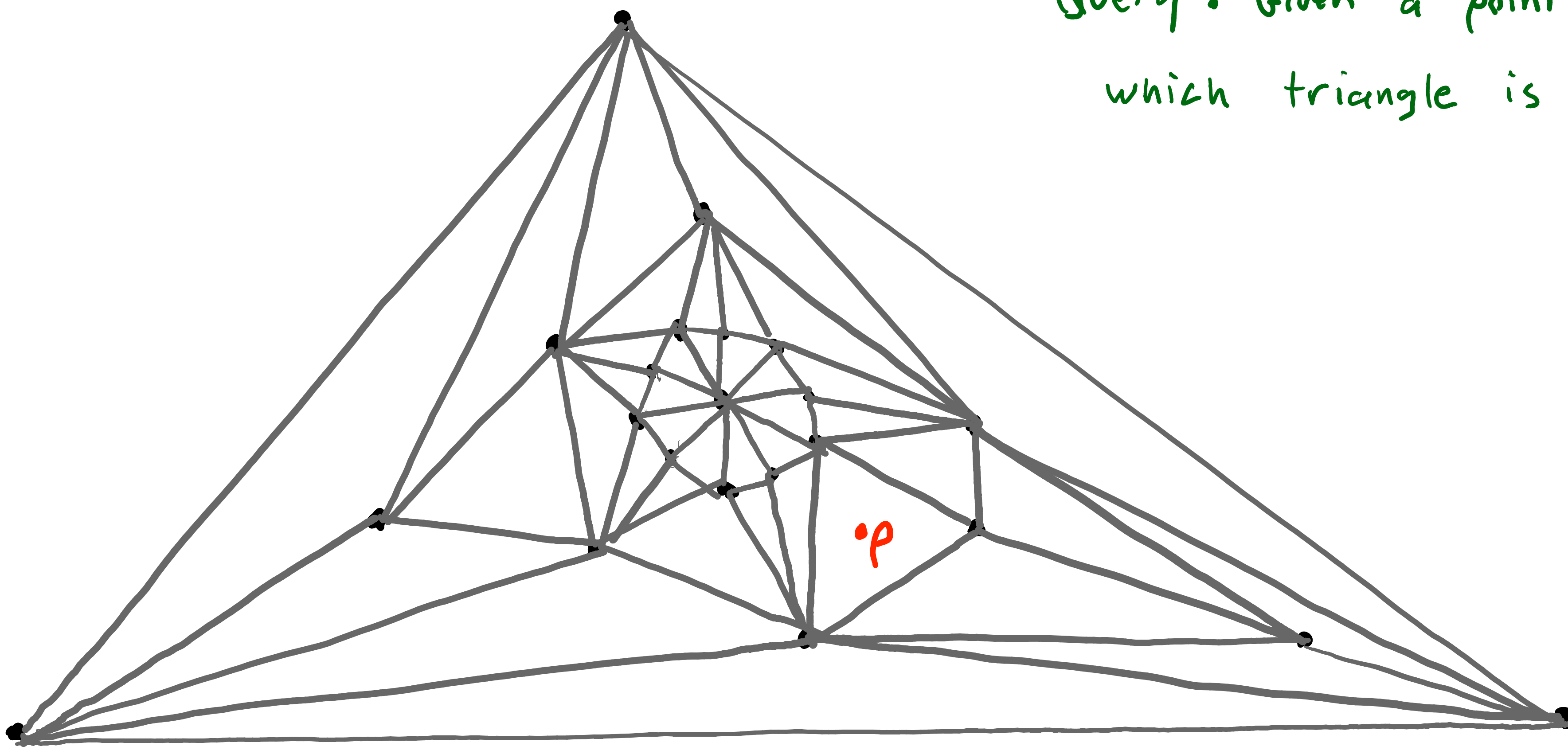

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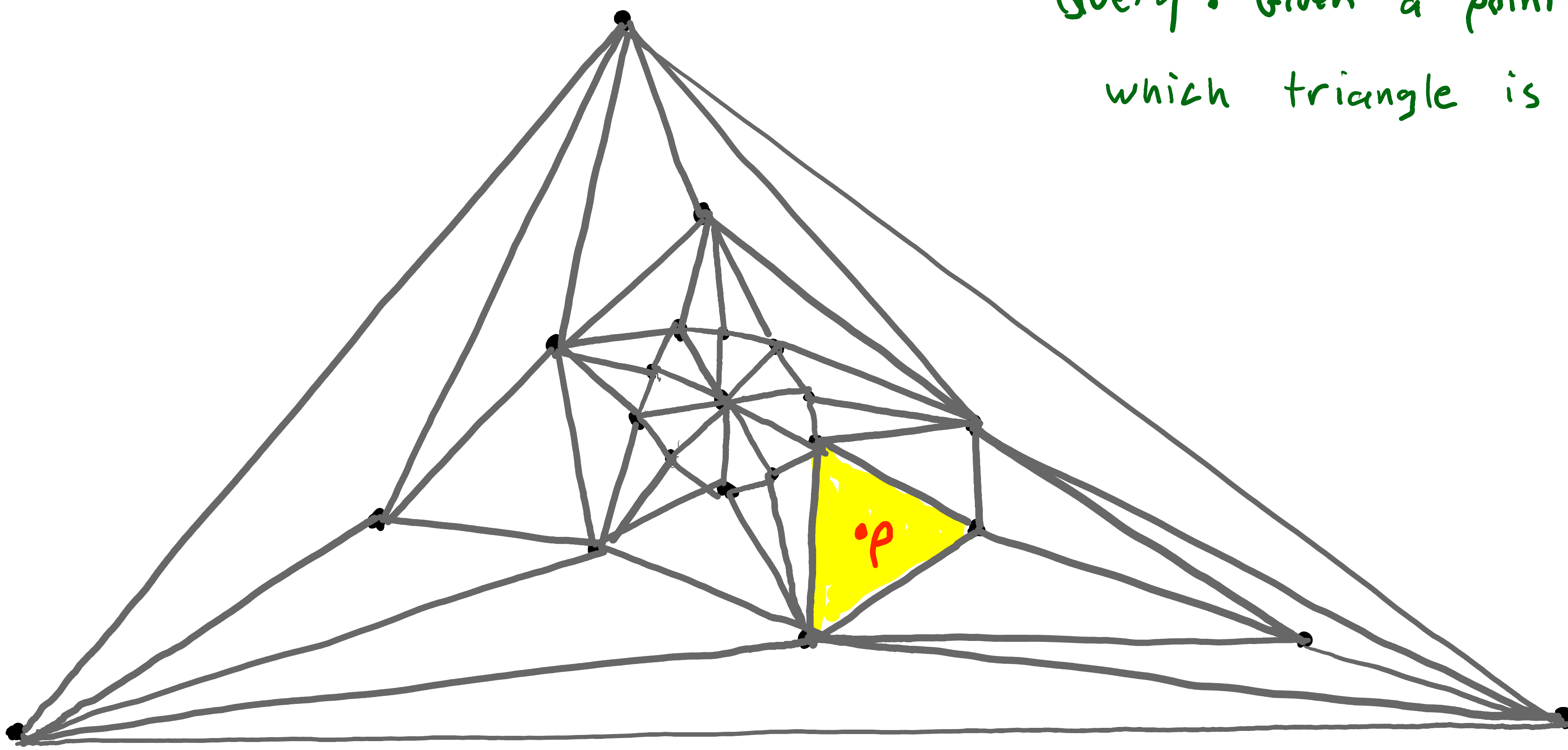

Planar Point Location

Input: A triangulation

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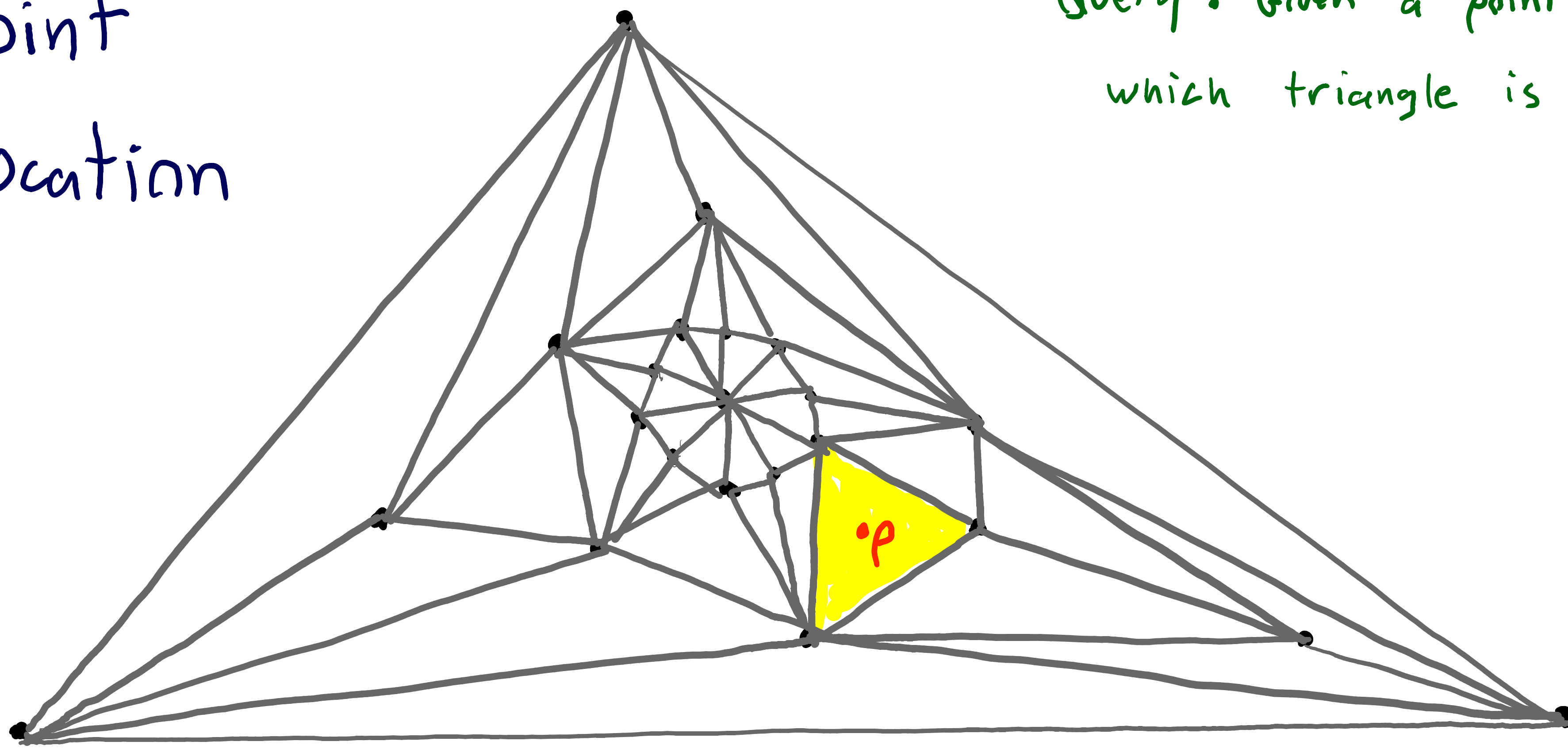

Basic Idea: Refinement

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Let's look at 1-D

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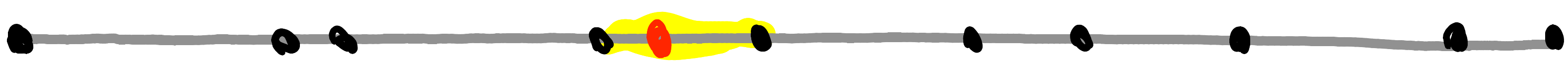

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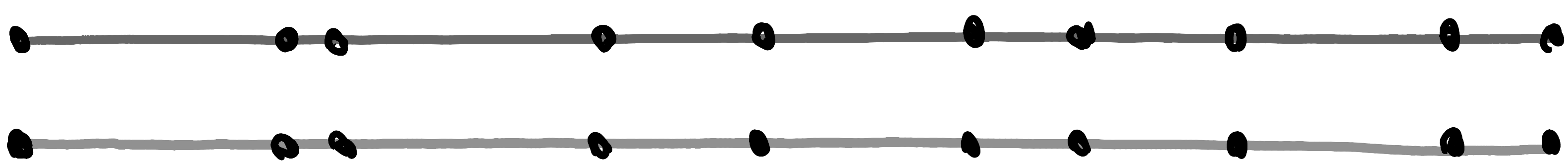

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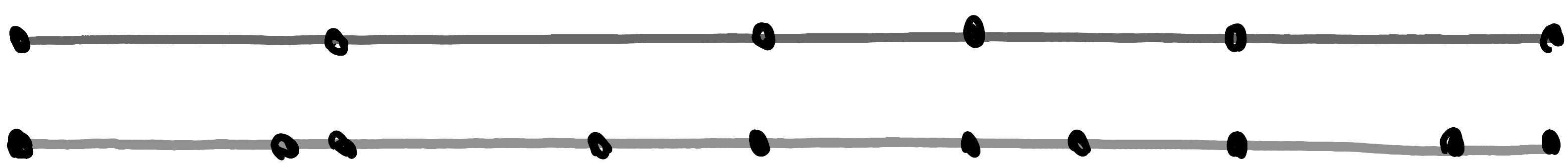

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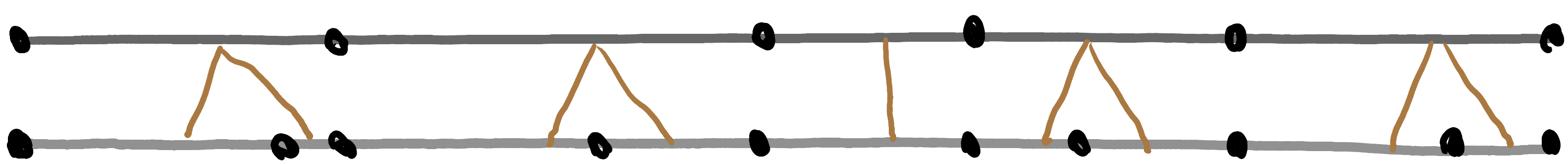

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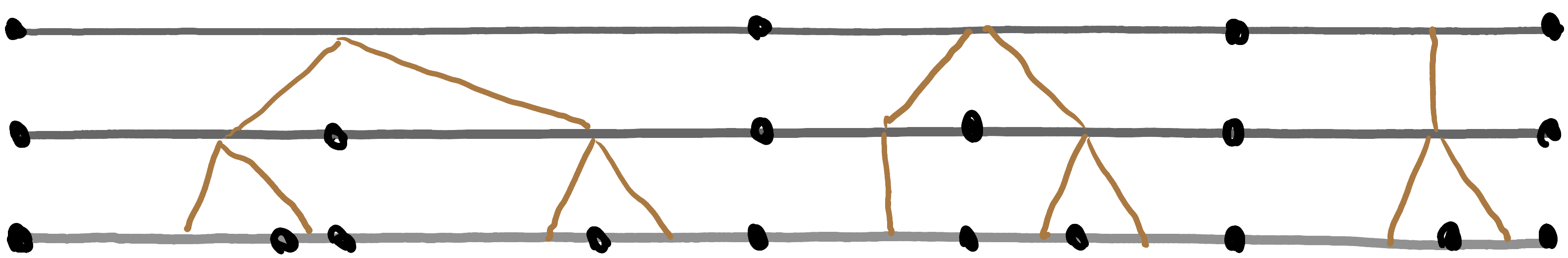

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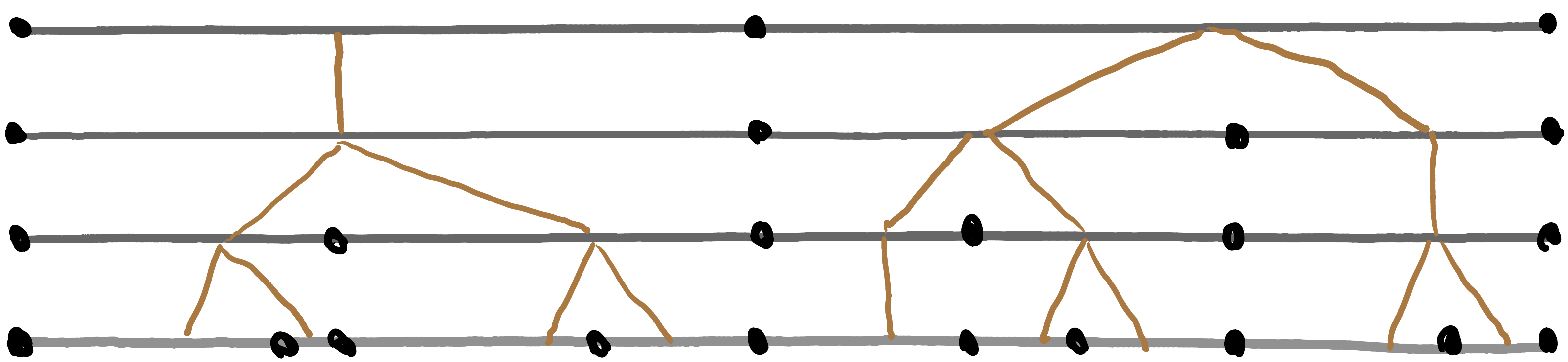

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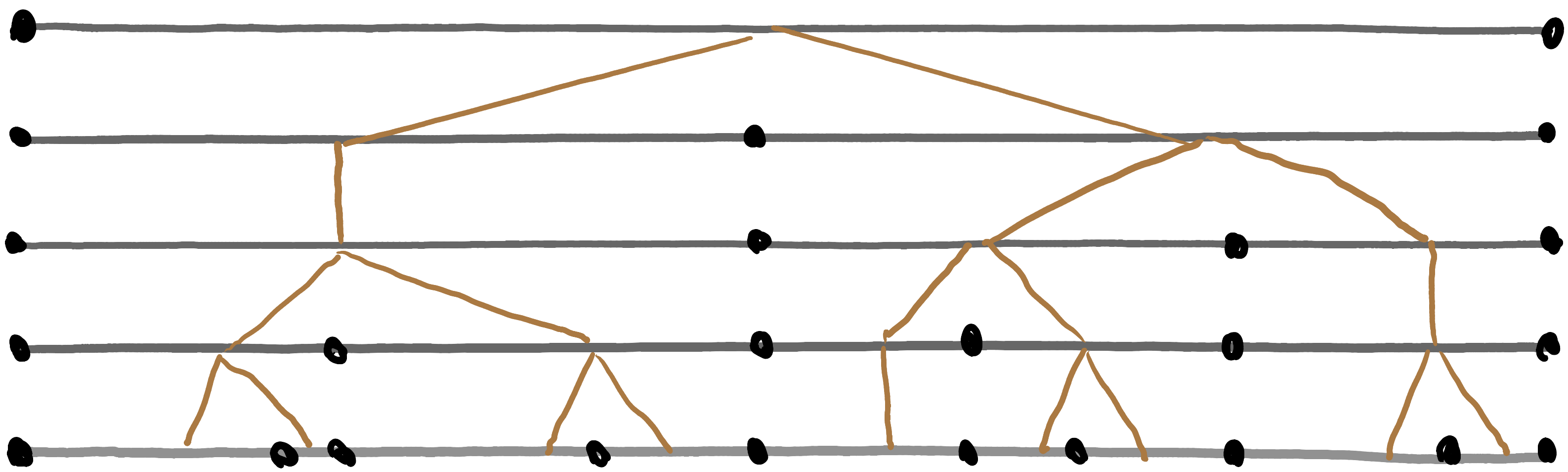

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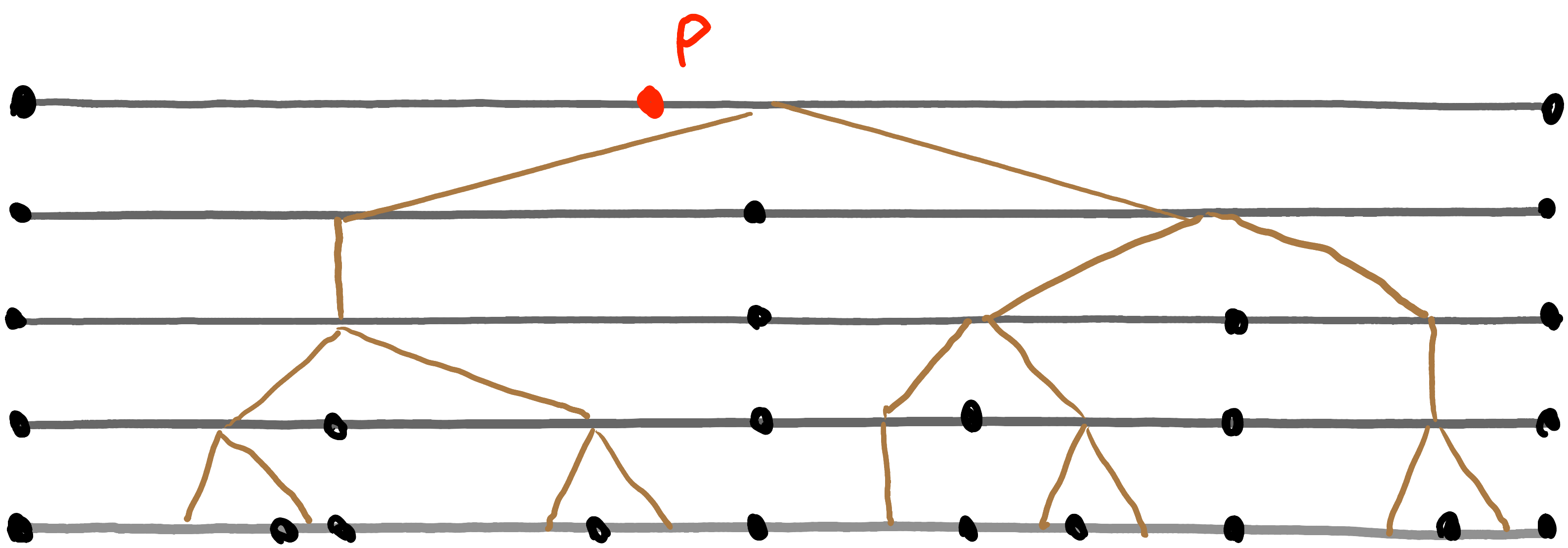

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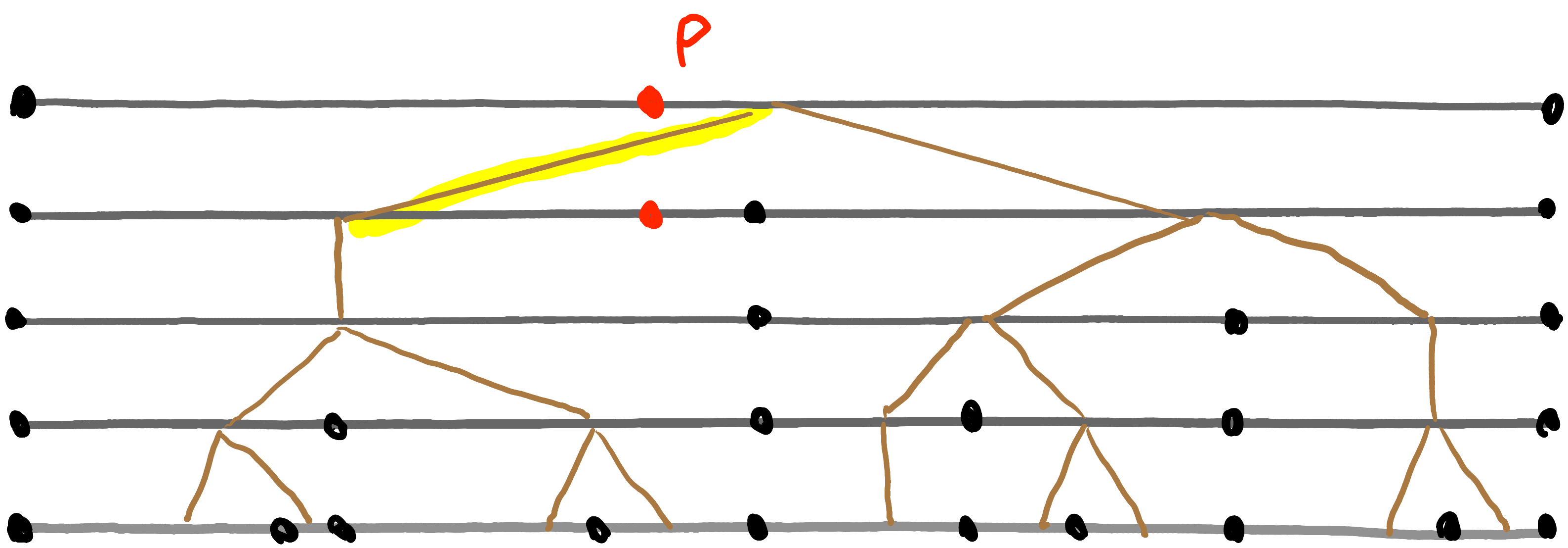

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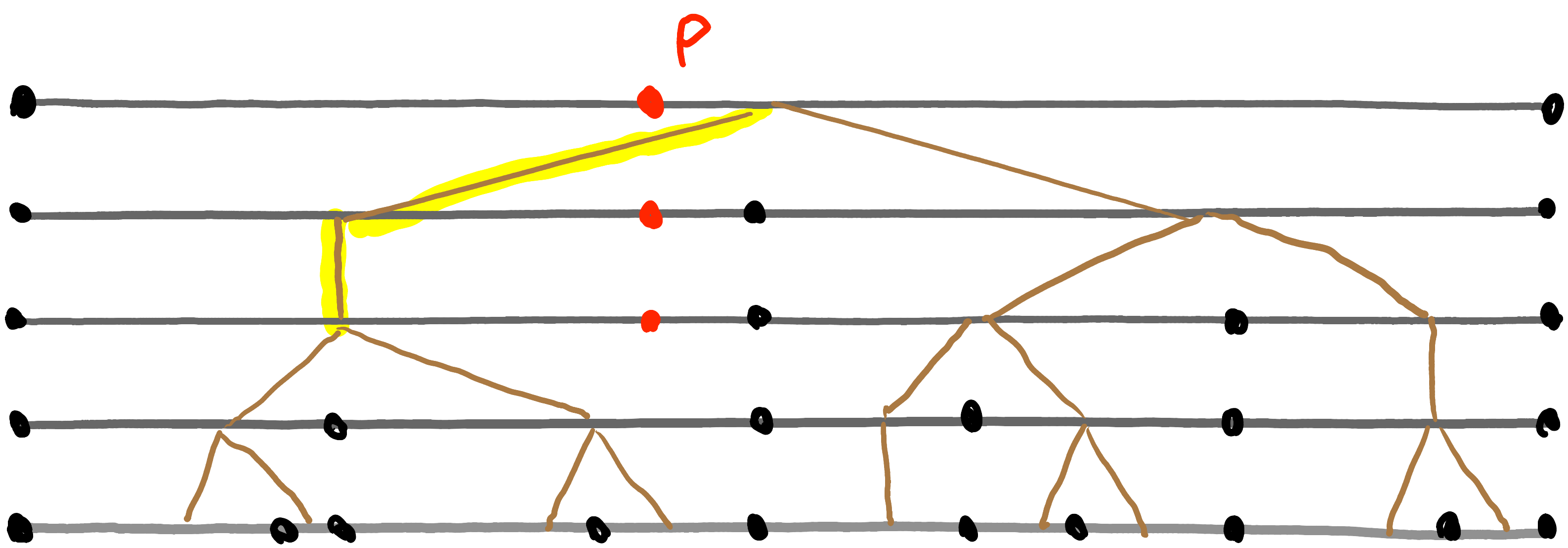

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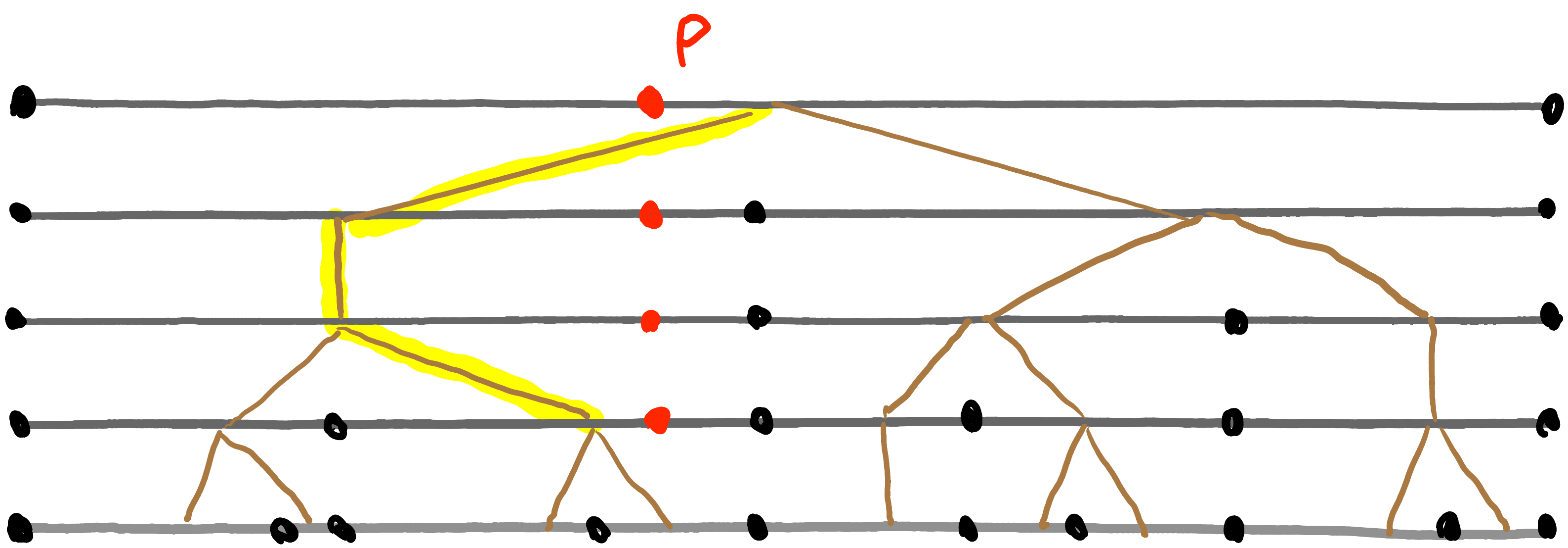


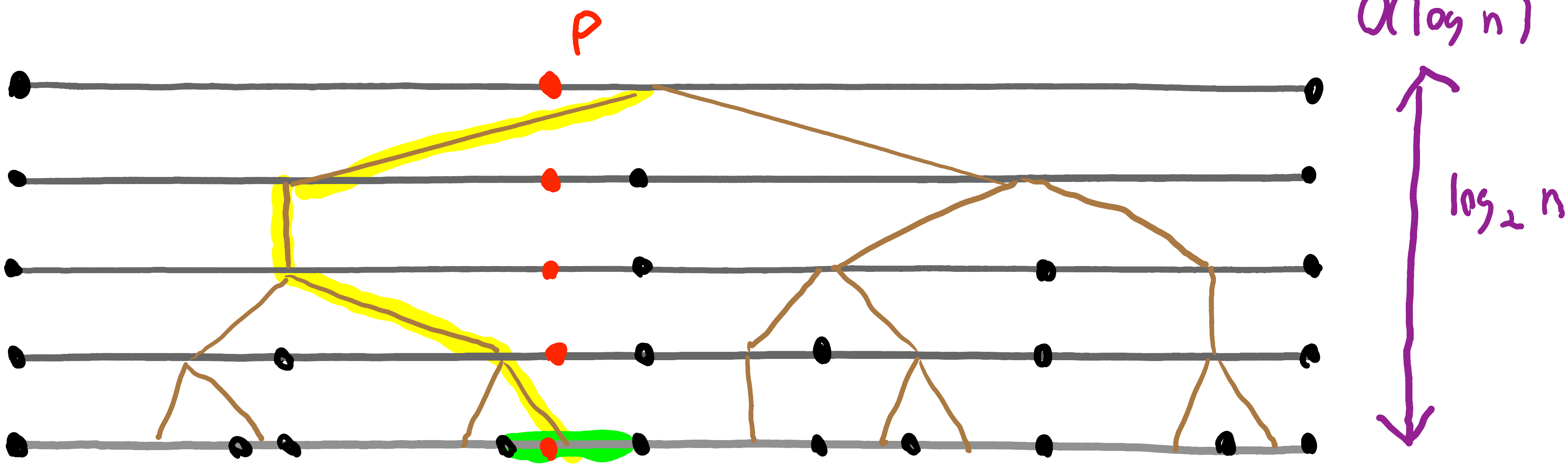
Analysis

Construction time: $O(n)$

$T(n) = T(\frac{n}{2}) + n$

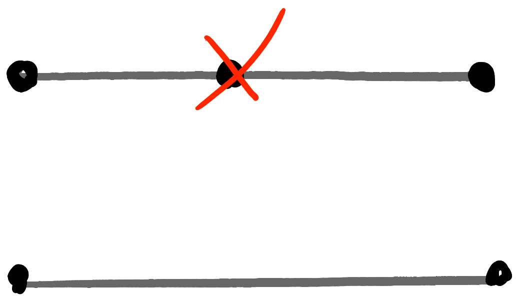
Query

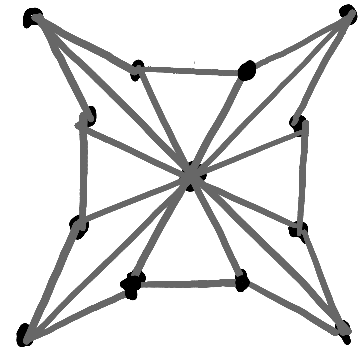
$O(\log n)$



1D $\rightsquigarrow$ 2D

1D ⟶ 2D





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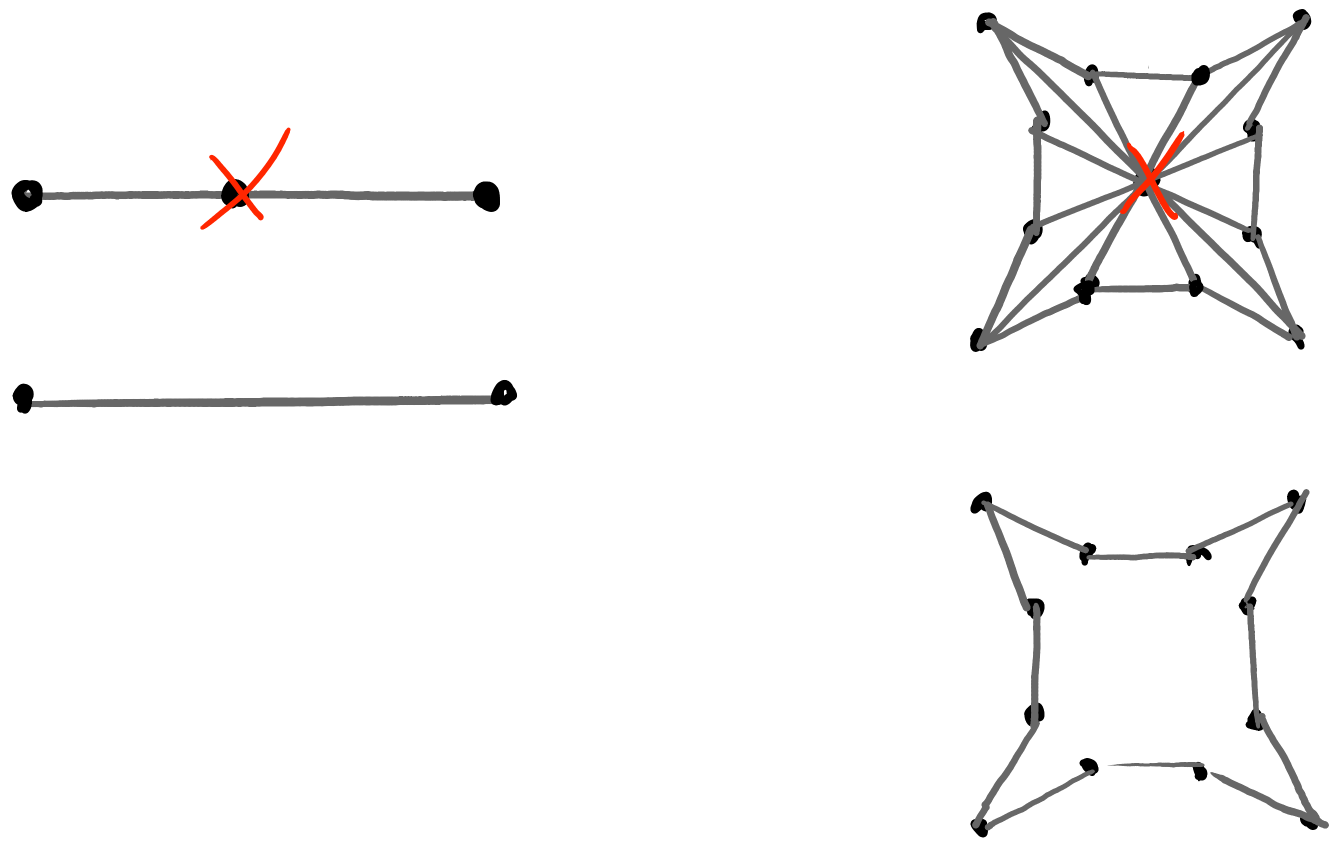

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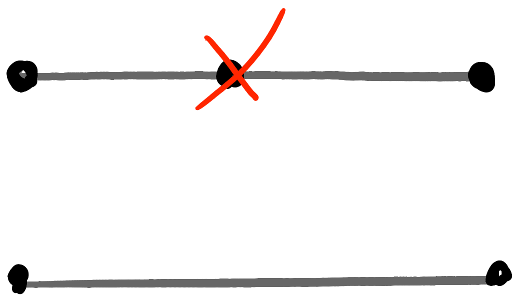
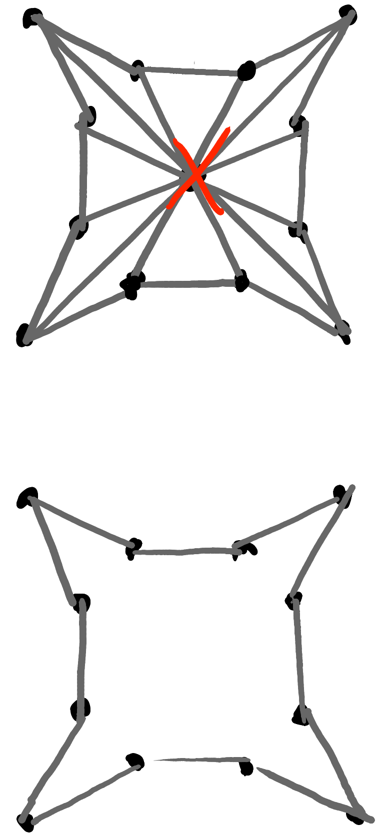
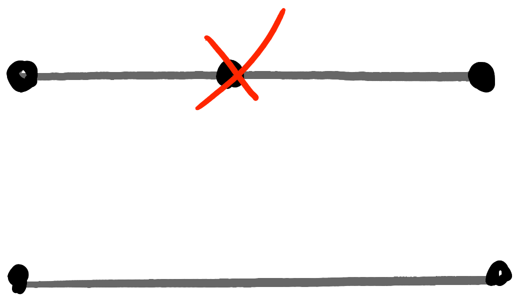
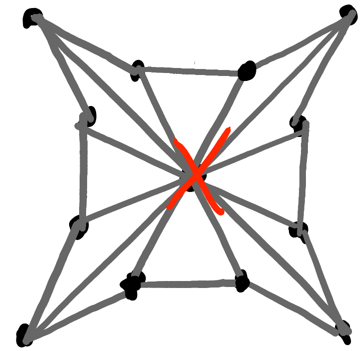




- Not a triangulation
- Possibly linear size

1D $\longrightarrow$ 2D





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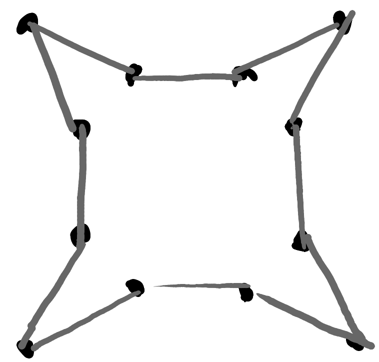

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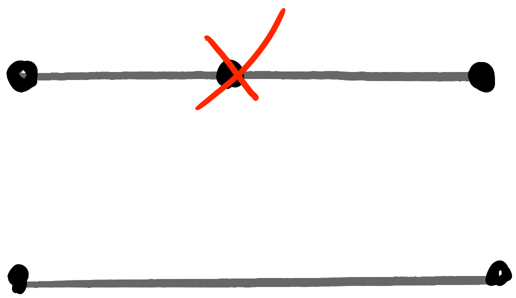

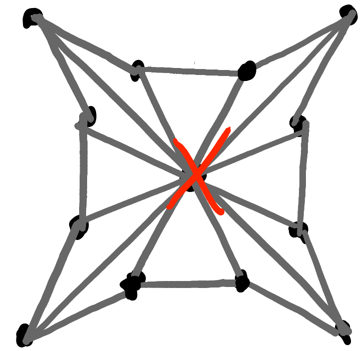

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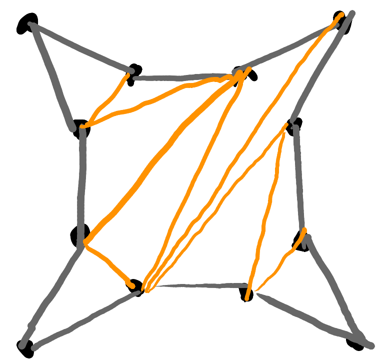

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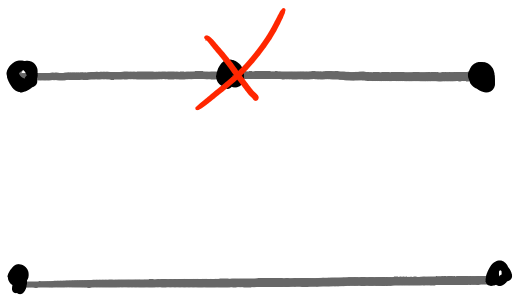

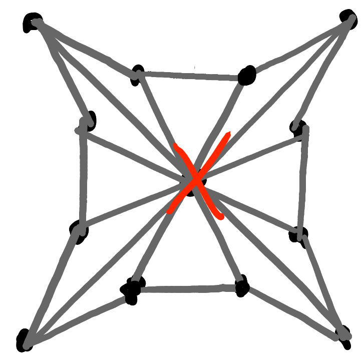

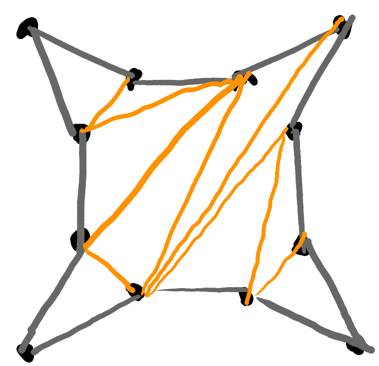

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- Most vertexes in a triangulation have small degree (will prove now)

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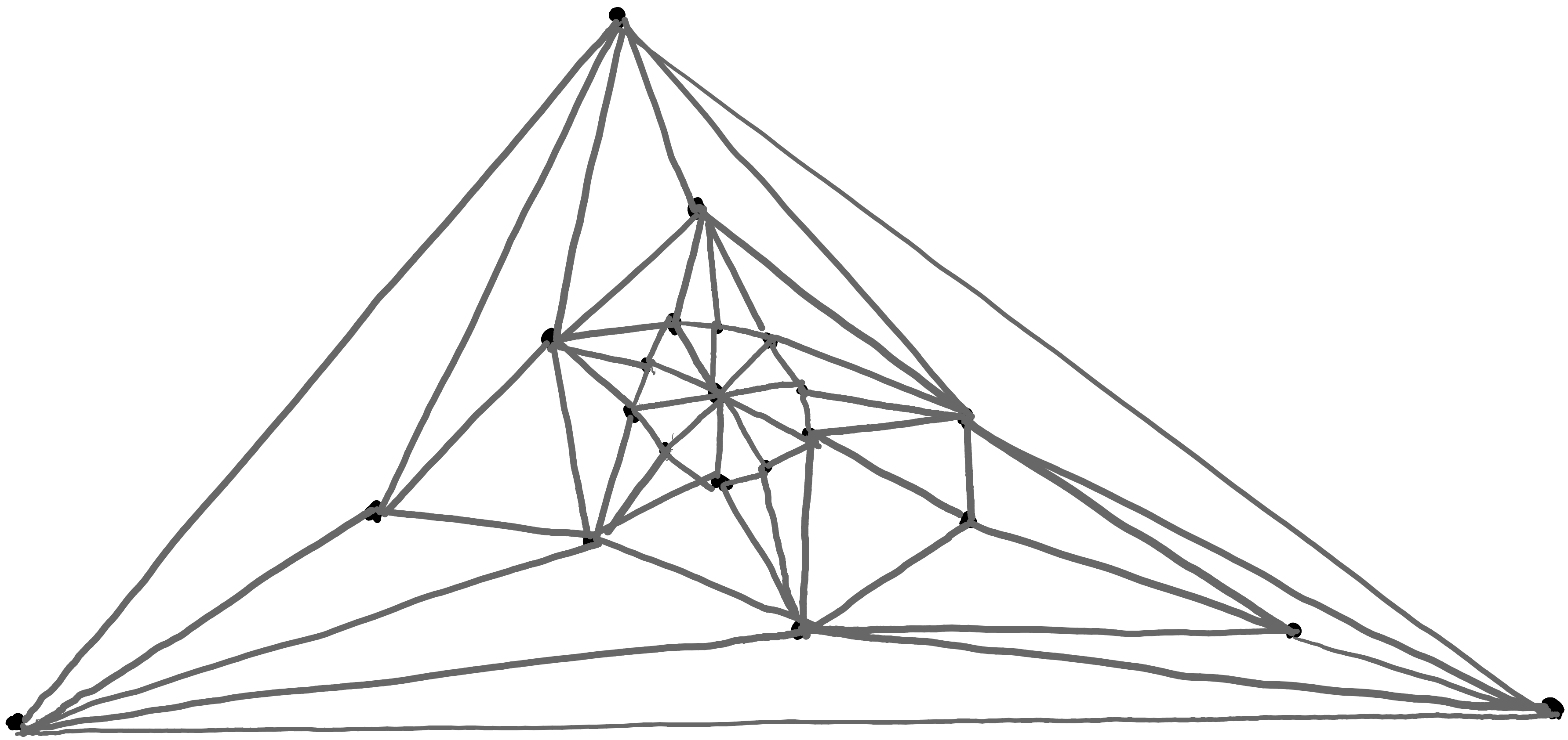
- $\geq$ Half of interior vertexes have degree 3, 4, 5, 6, 7, 8

I see 17 interior vertexes, can I remove 8 with degree ≤8?



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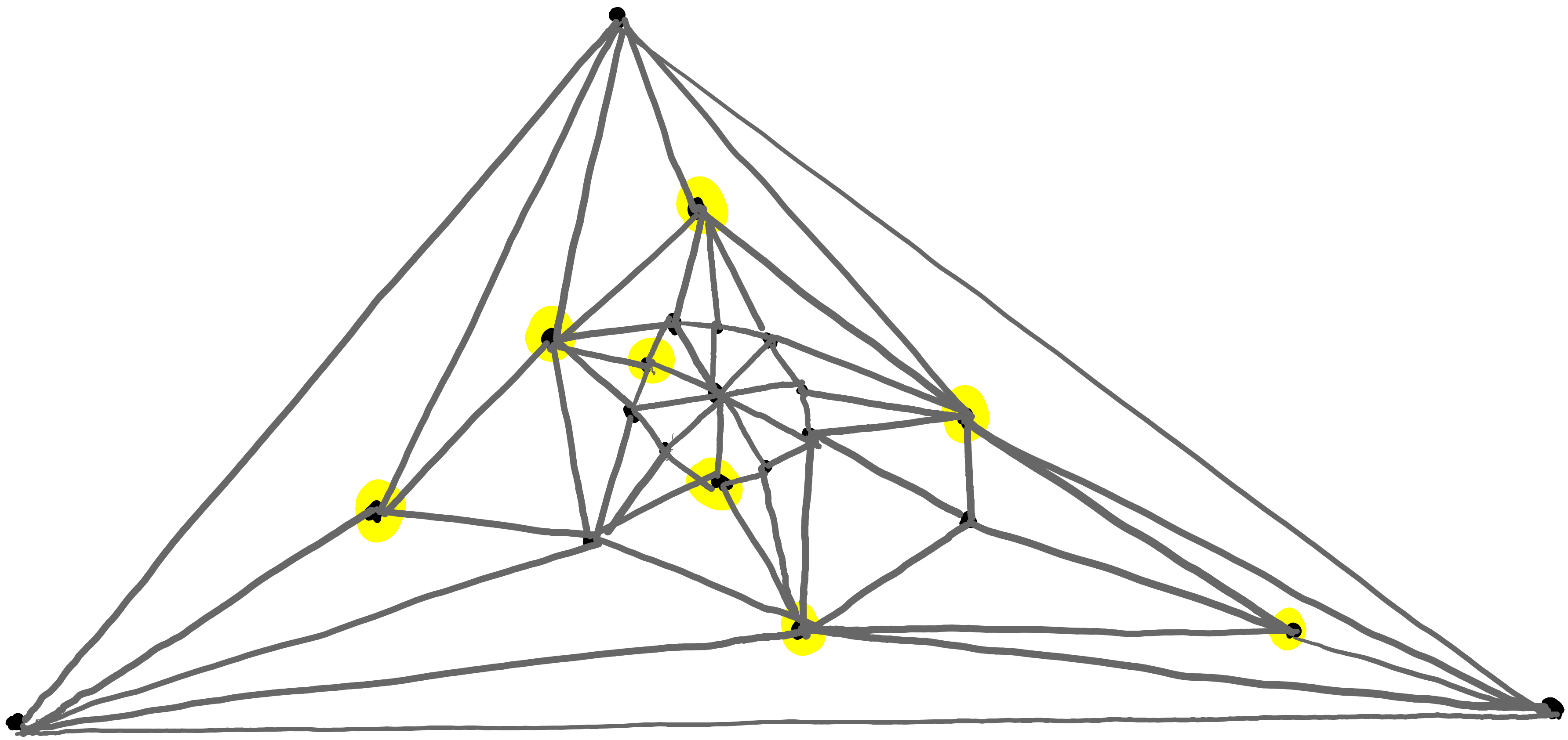

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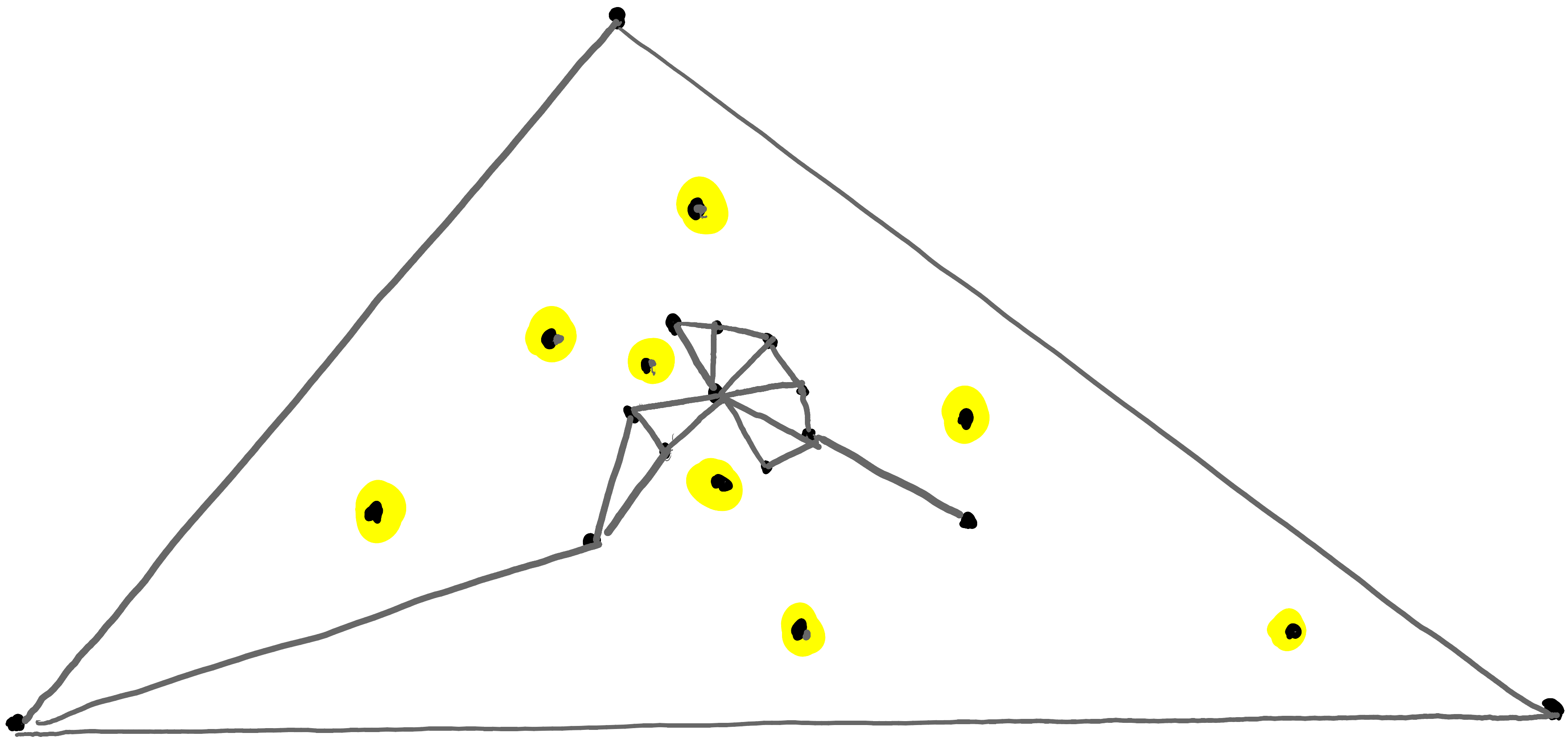

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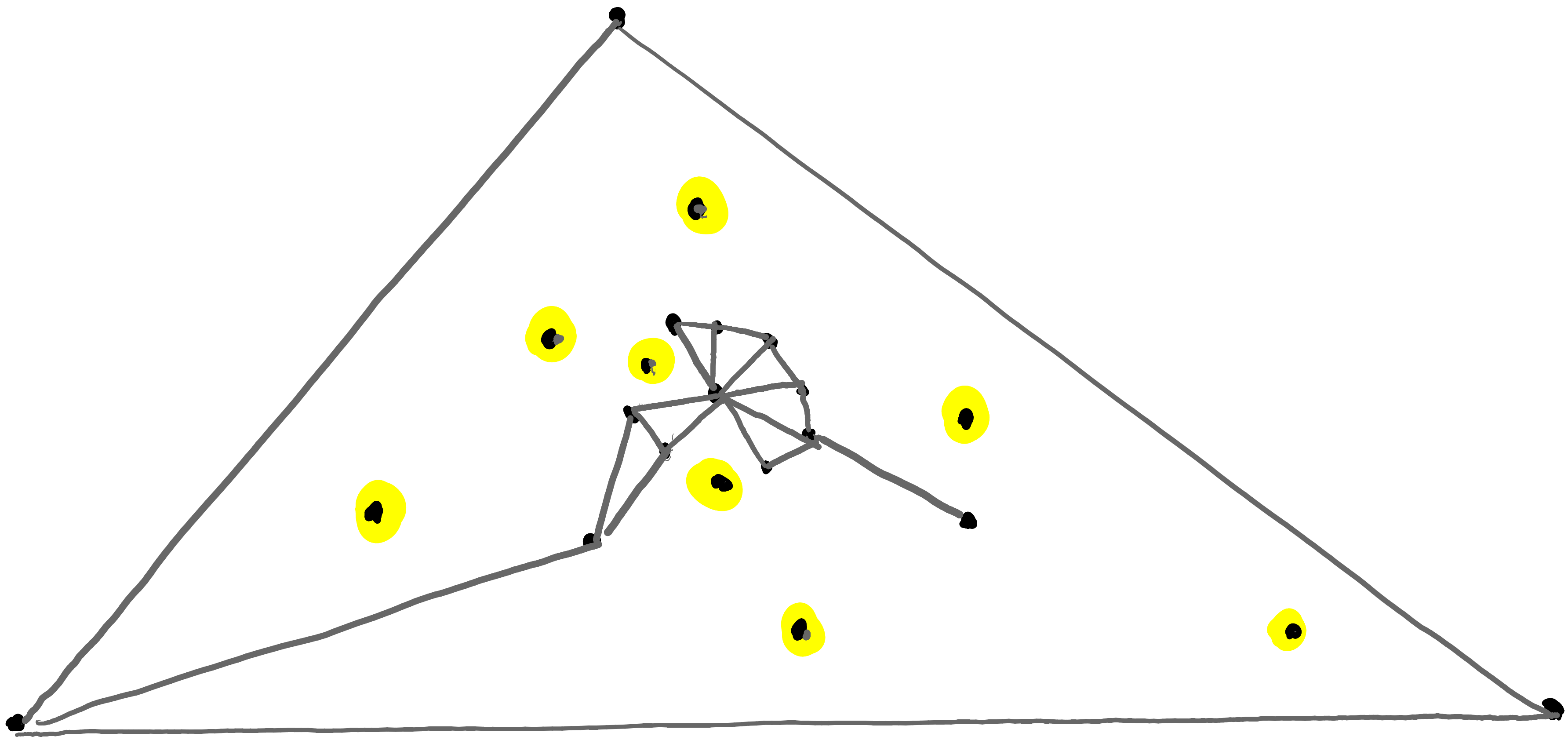
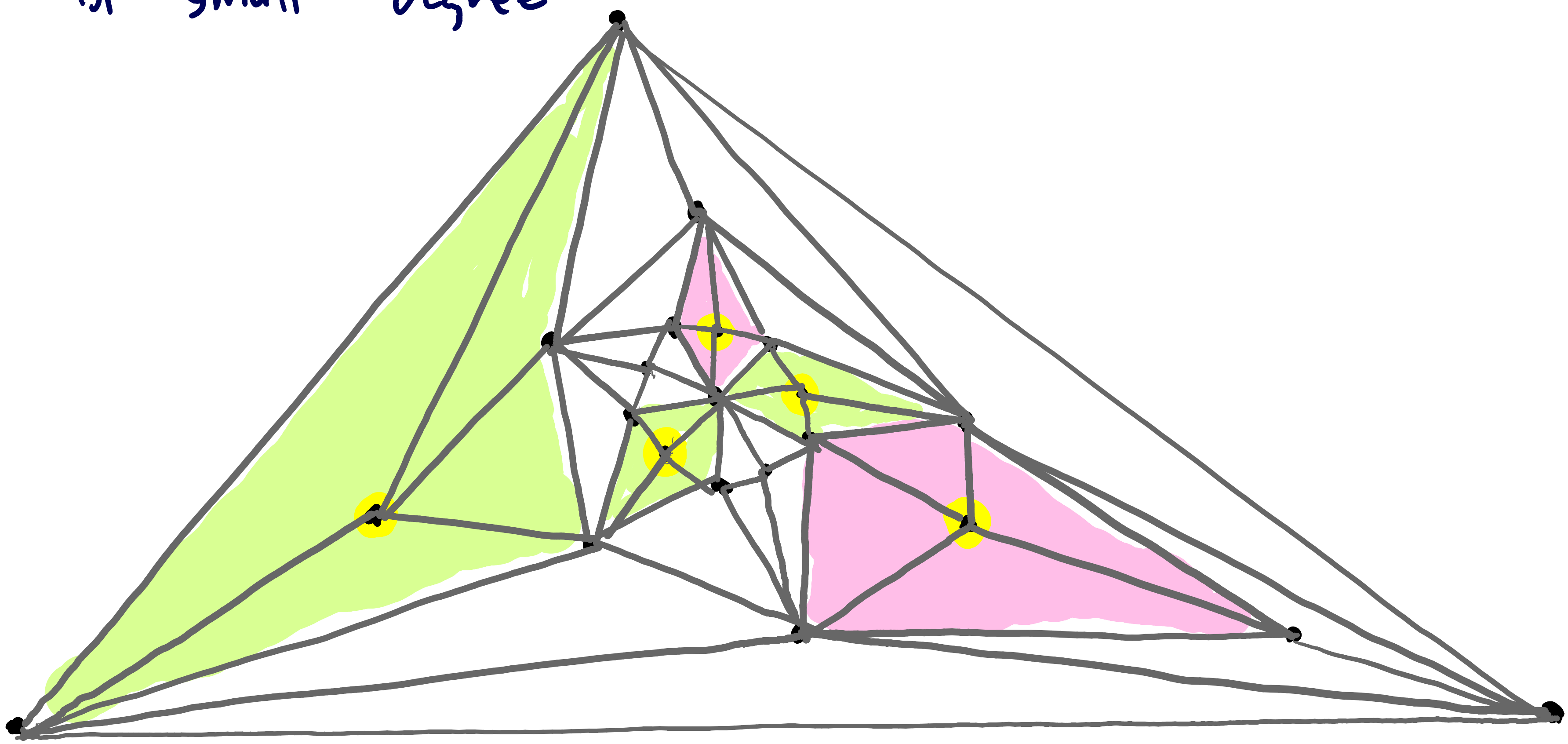
No! This is not a refinement, this is total destruction!



Remove an independent set of vertexes of small degree



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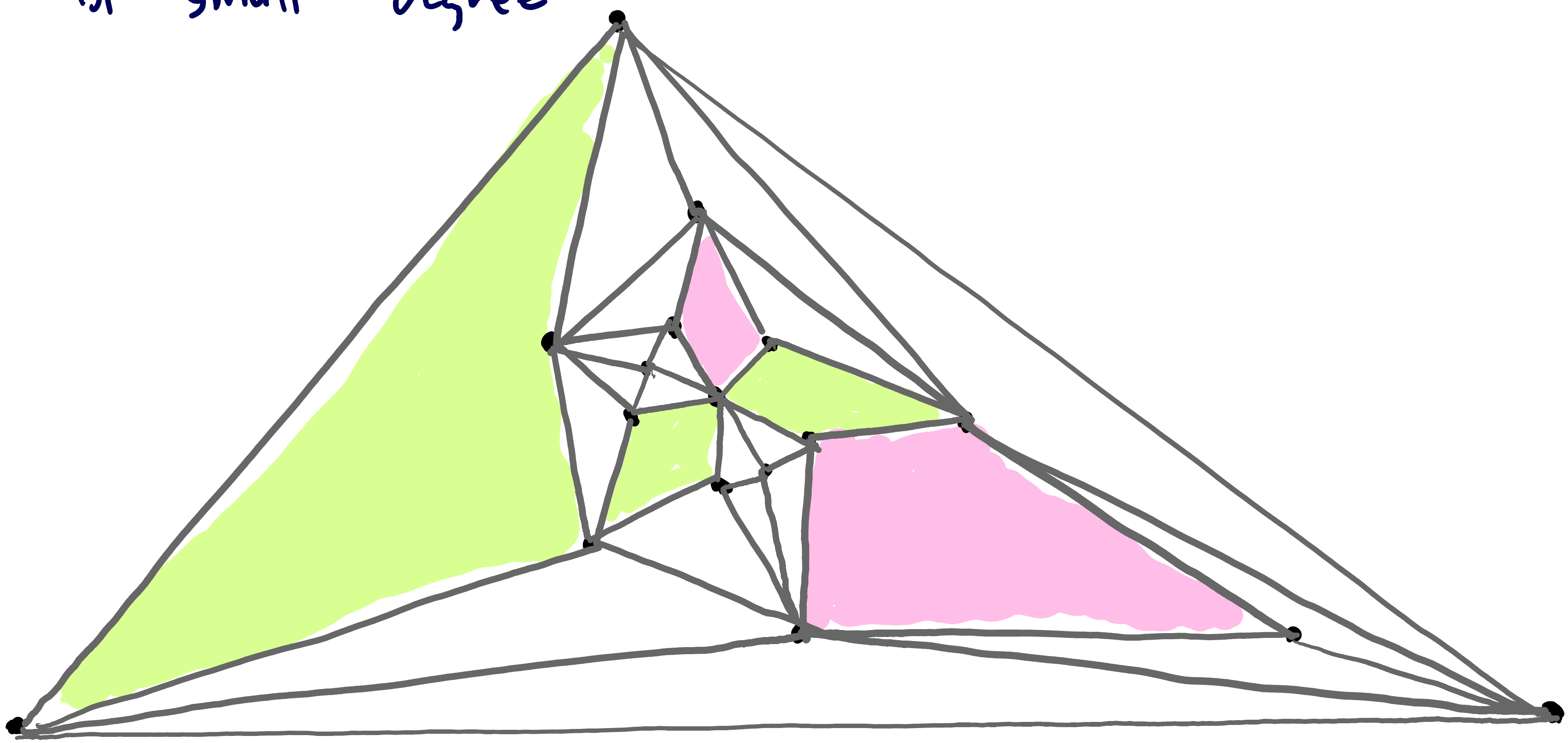

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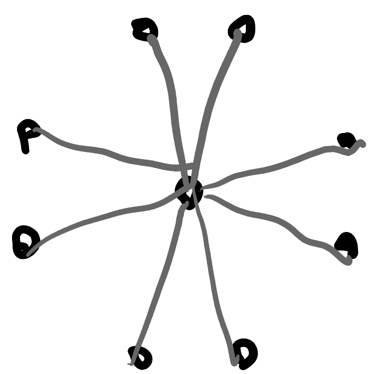

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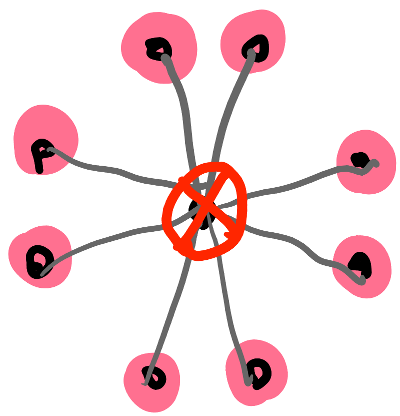

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Mark adjacent nodes as unremovable.

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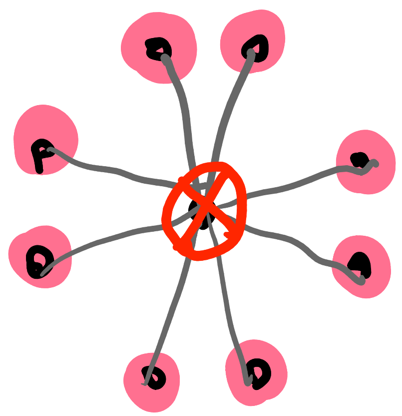

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$$\frac{1}{9} \cdot \frac{1}{2} = \frac{1}{18}$$ of nodes will be removed

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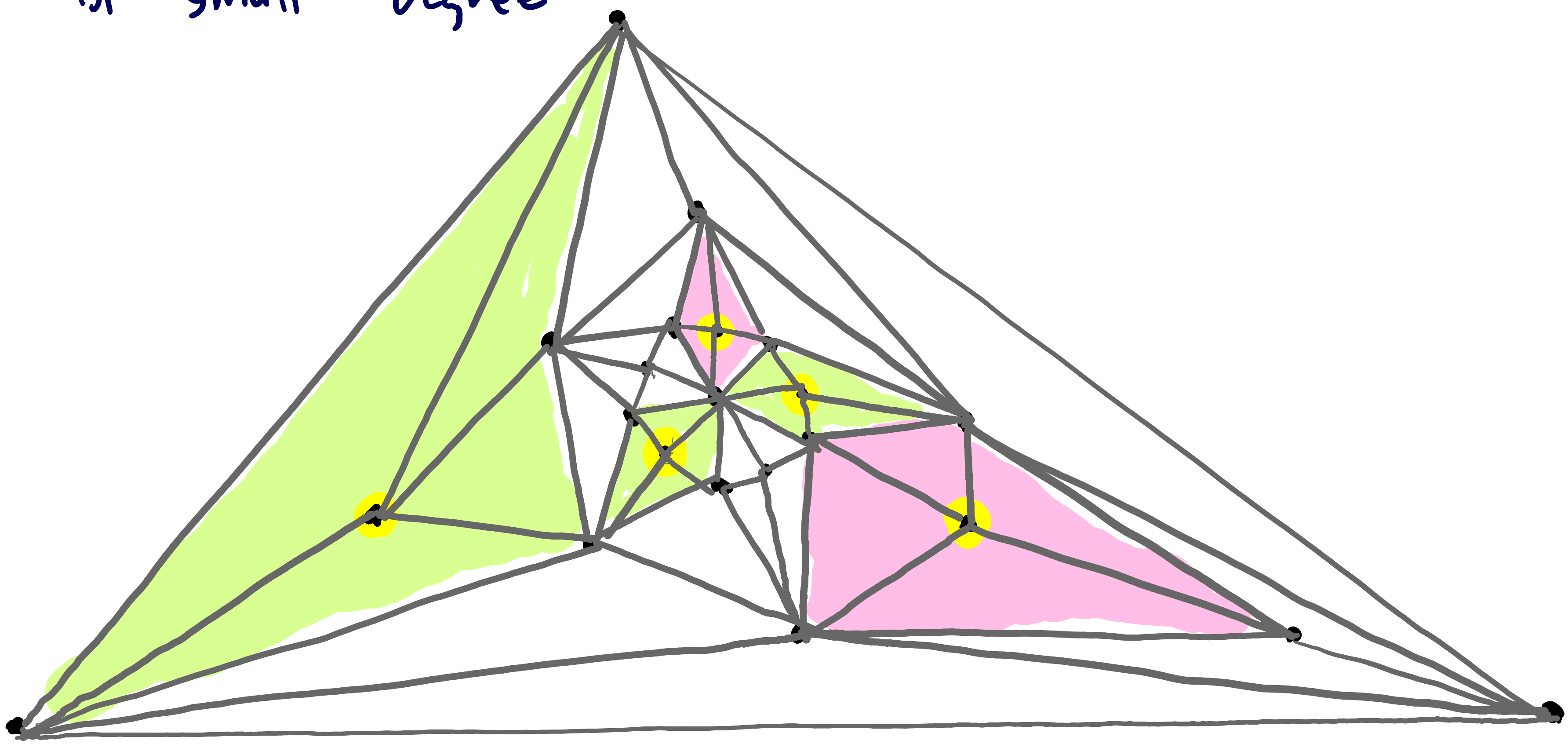

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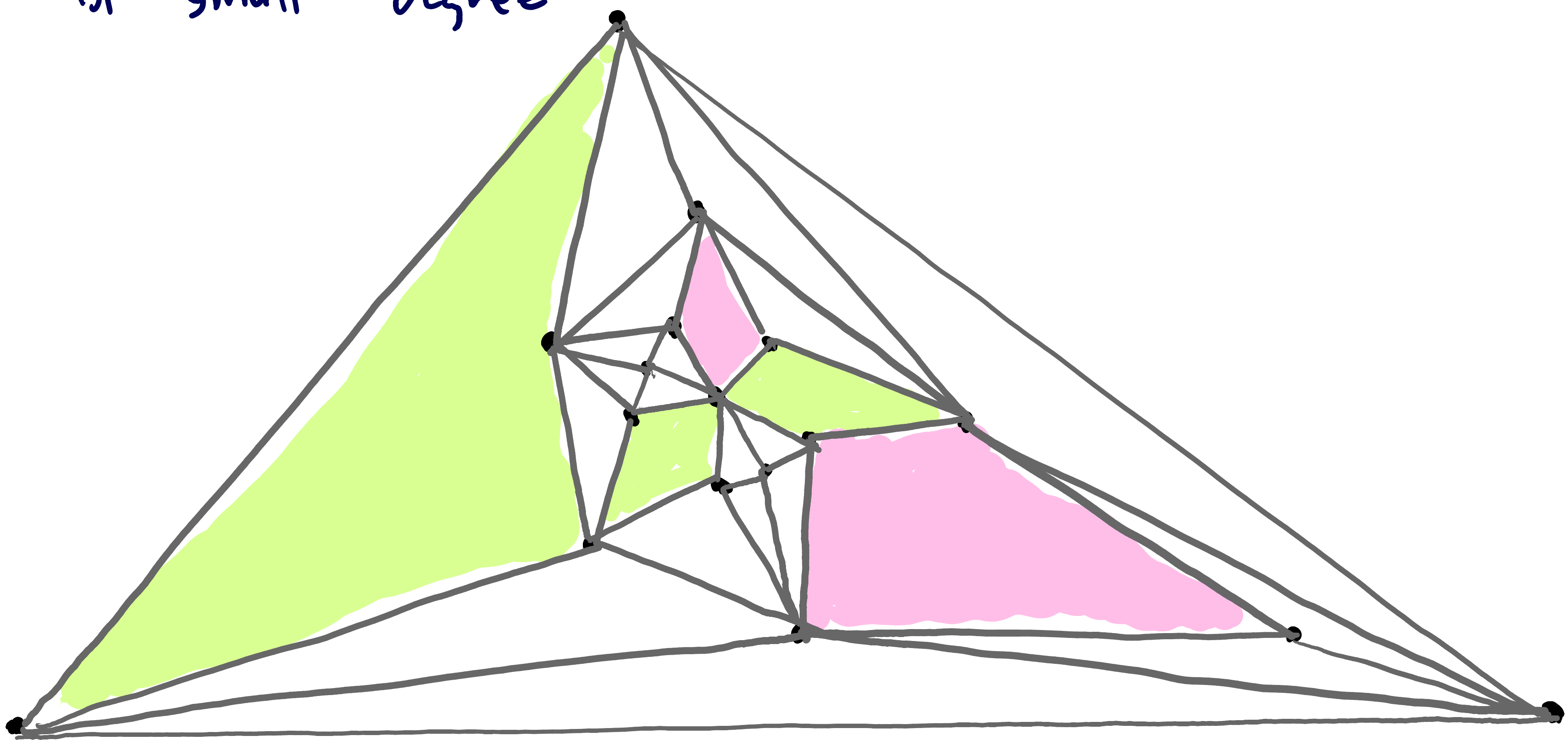

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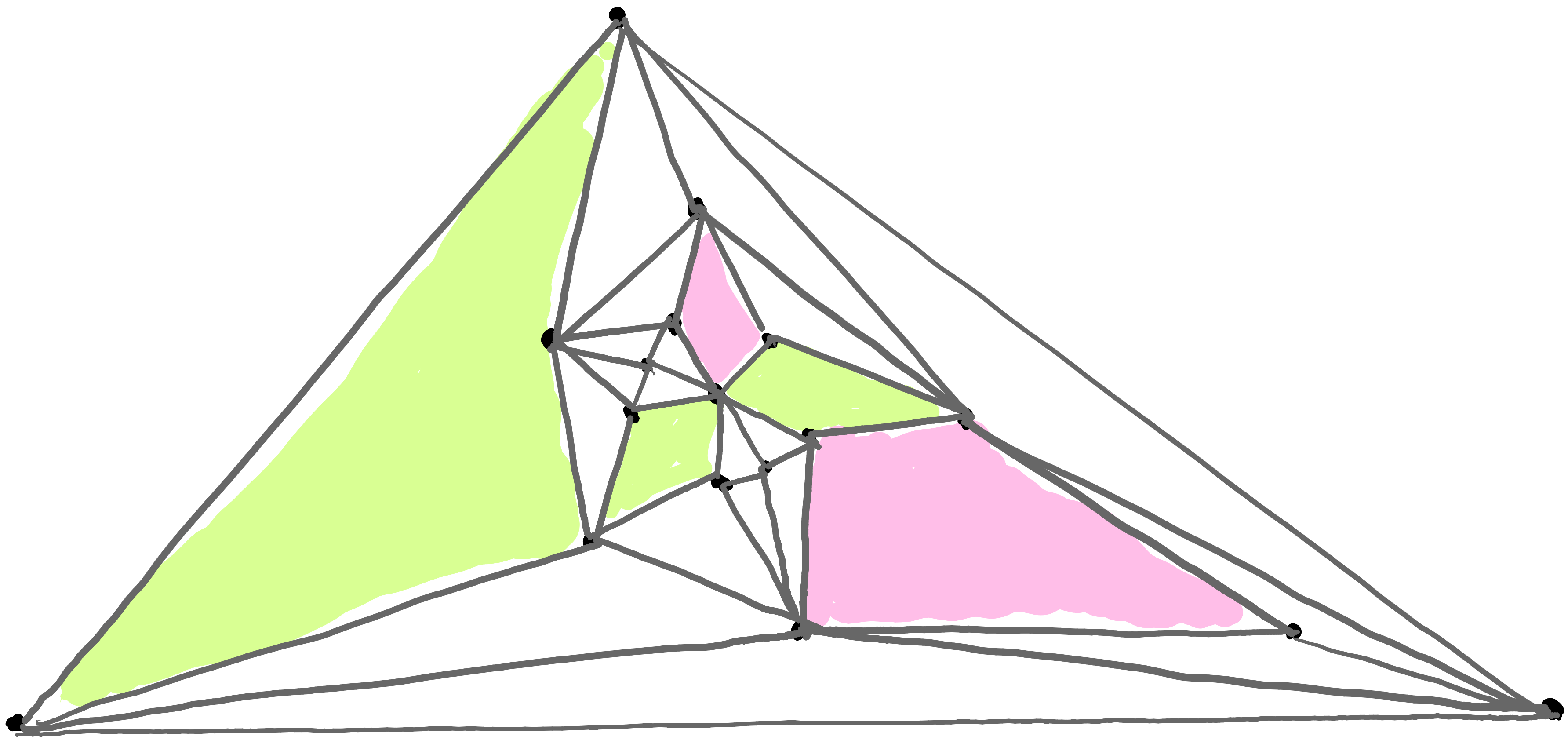

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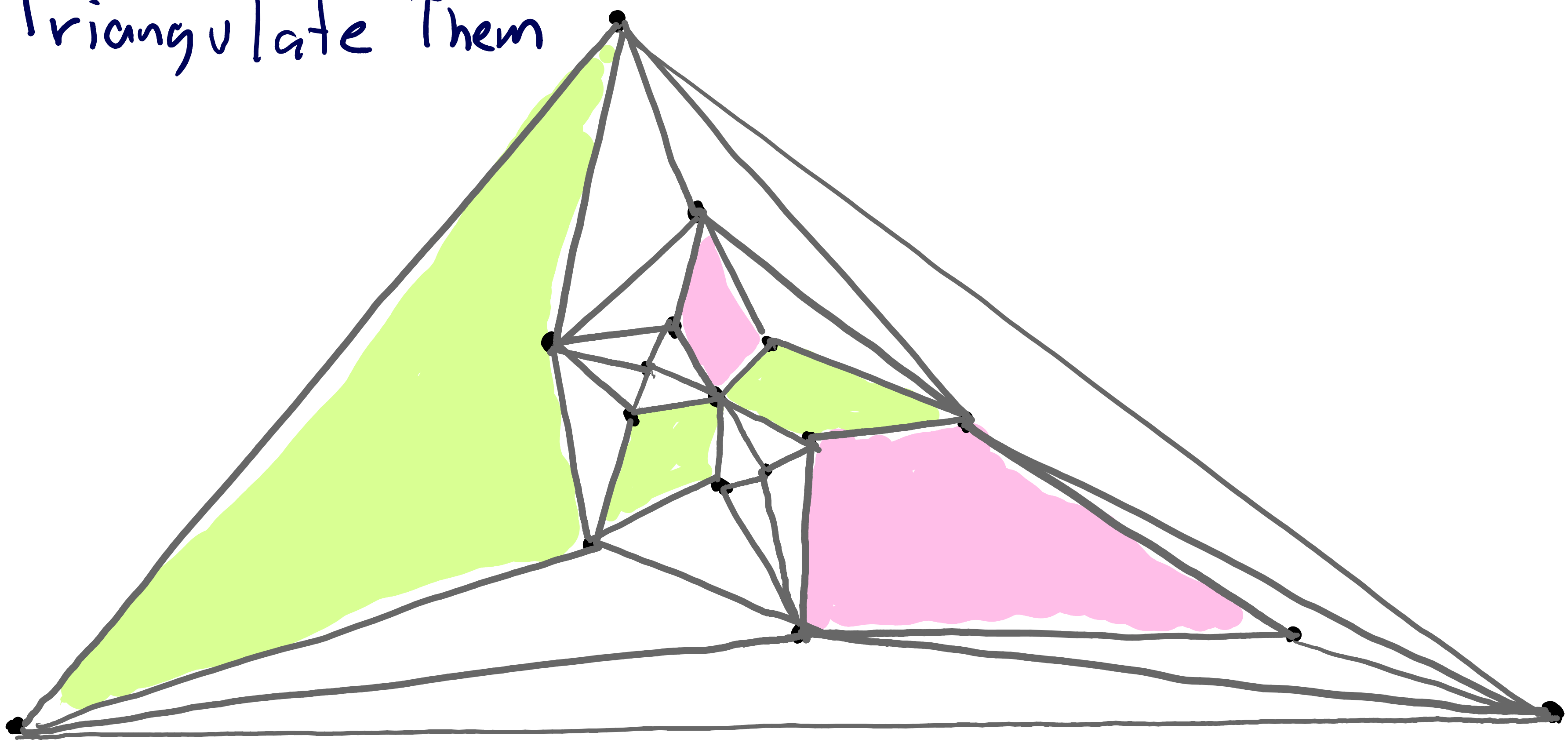

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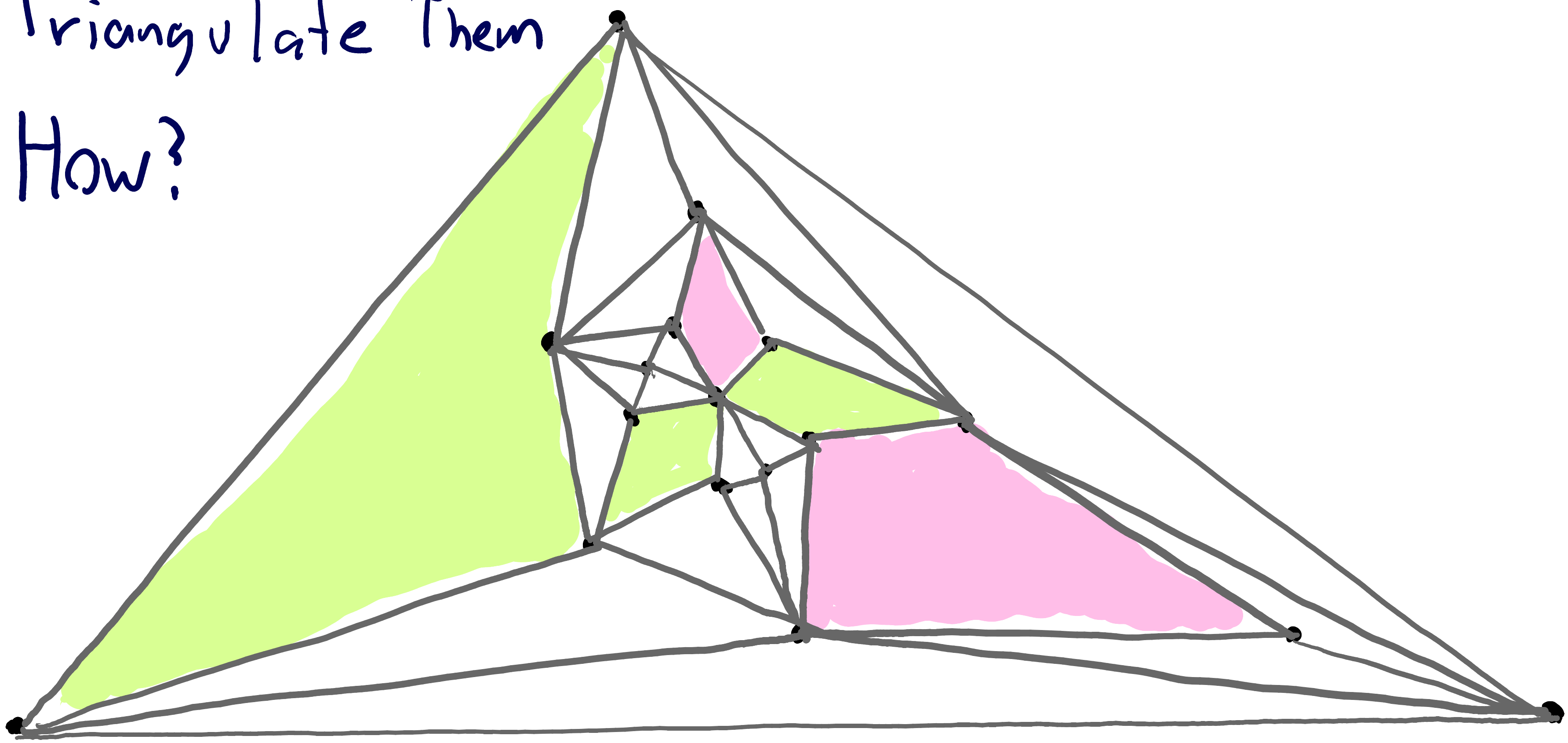

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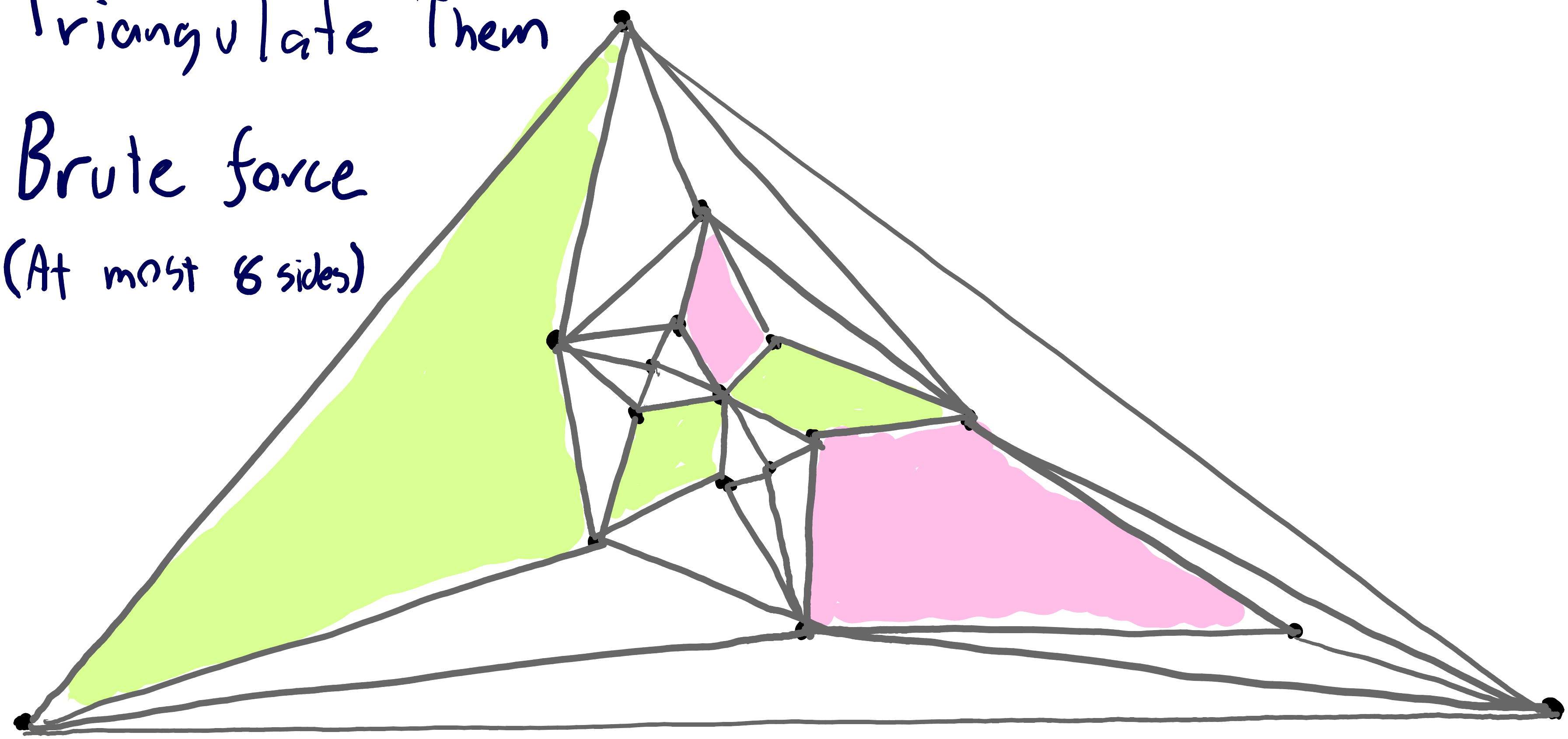
Brute force

(At most 8 sides)



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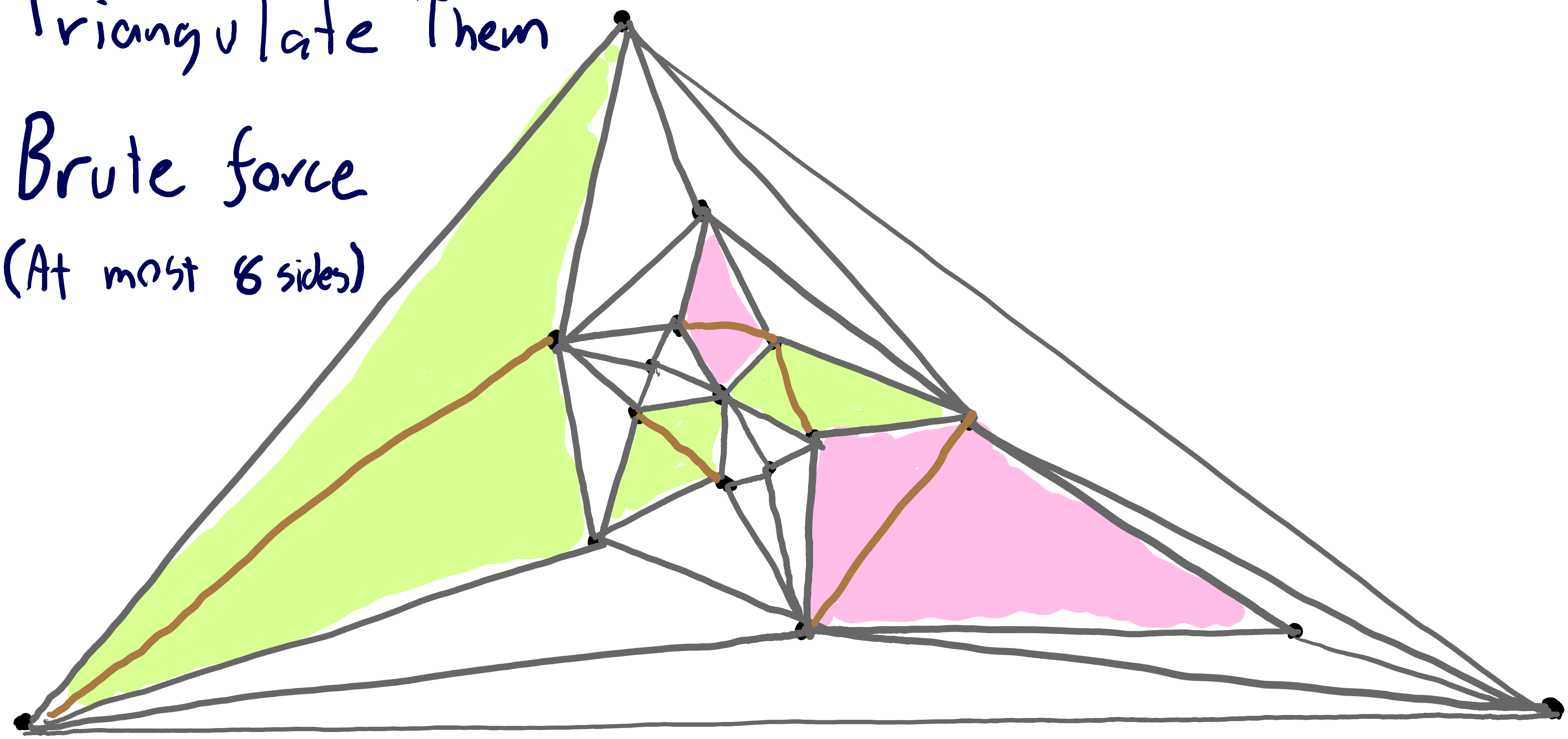

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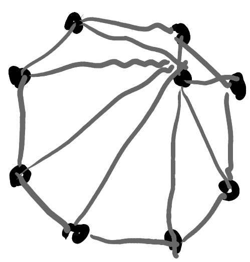

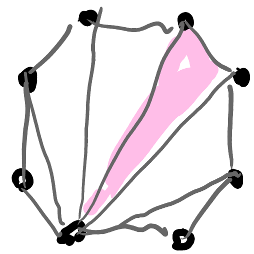

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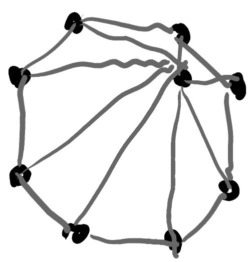
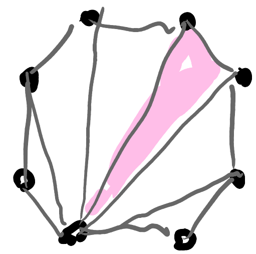




(4) Put a pointer from every triange in the OT to every triangle in the NT it overlaps.

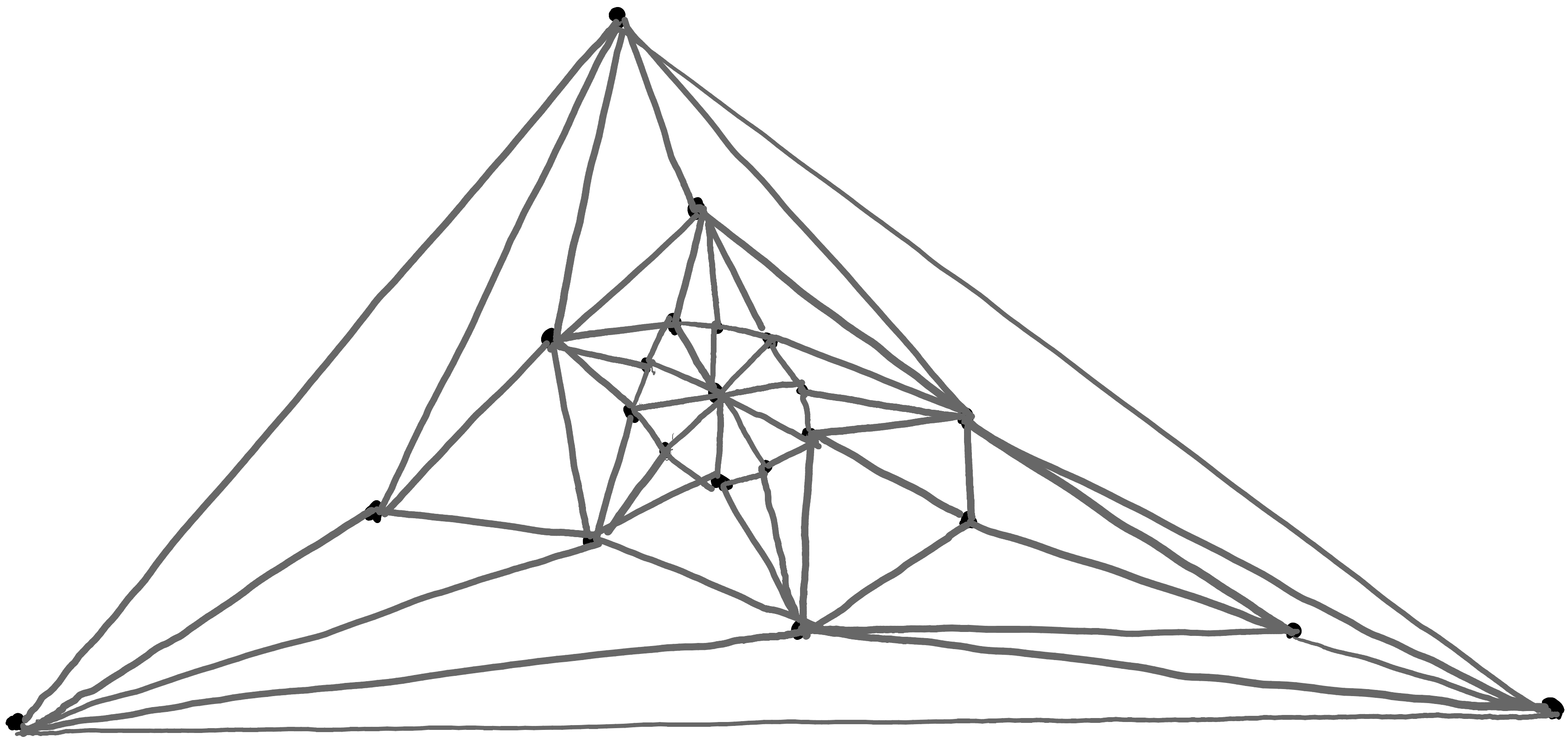
Then given a point and the triange in the OT which it is in the triangle in NT that it is in can be determined in $O(1)$.

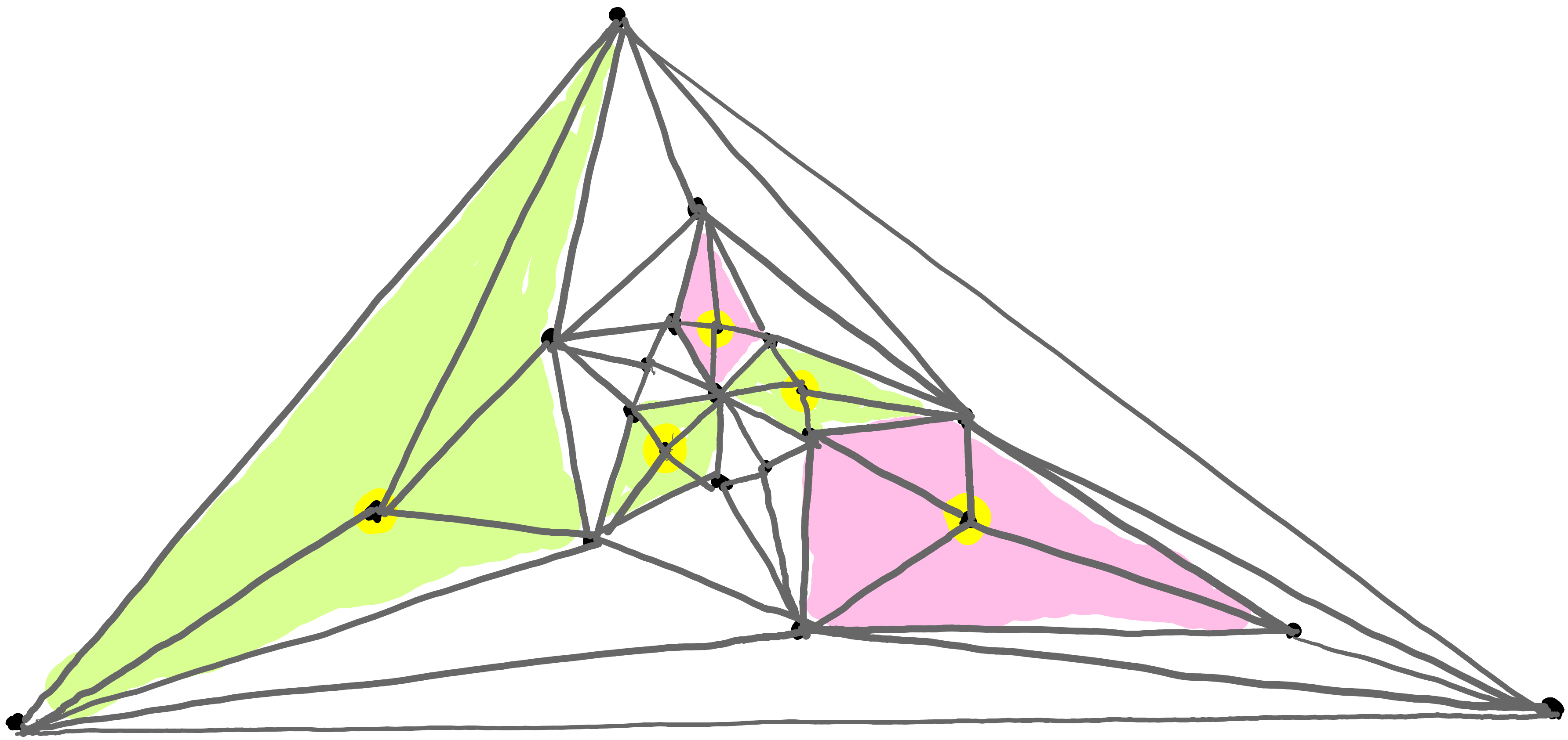
This gives you everything you need

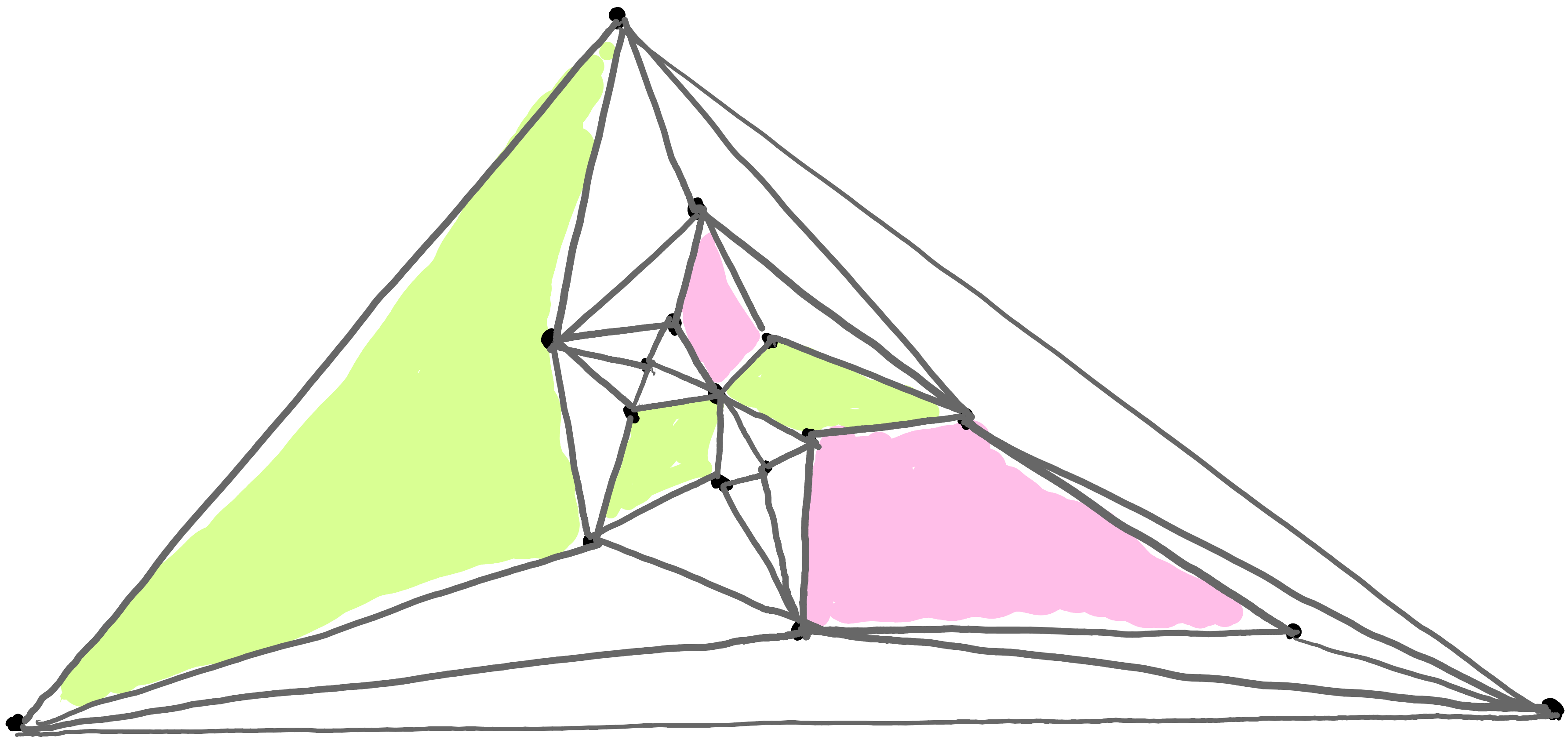
Just repeat the

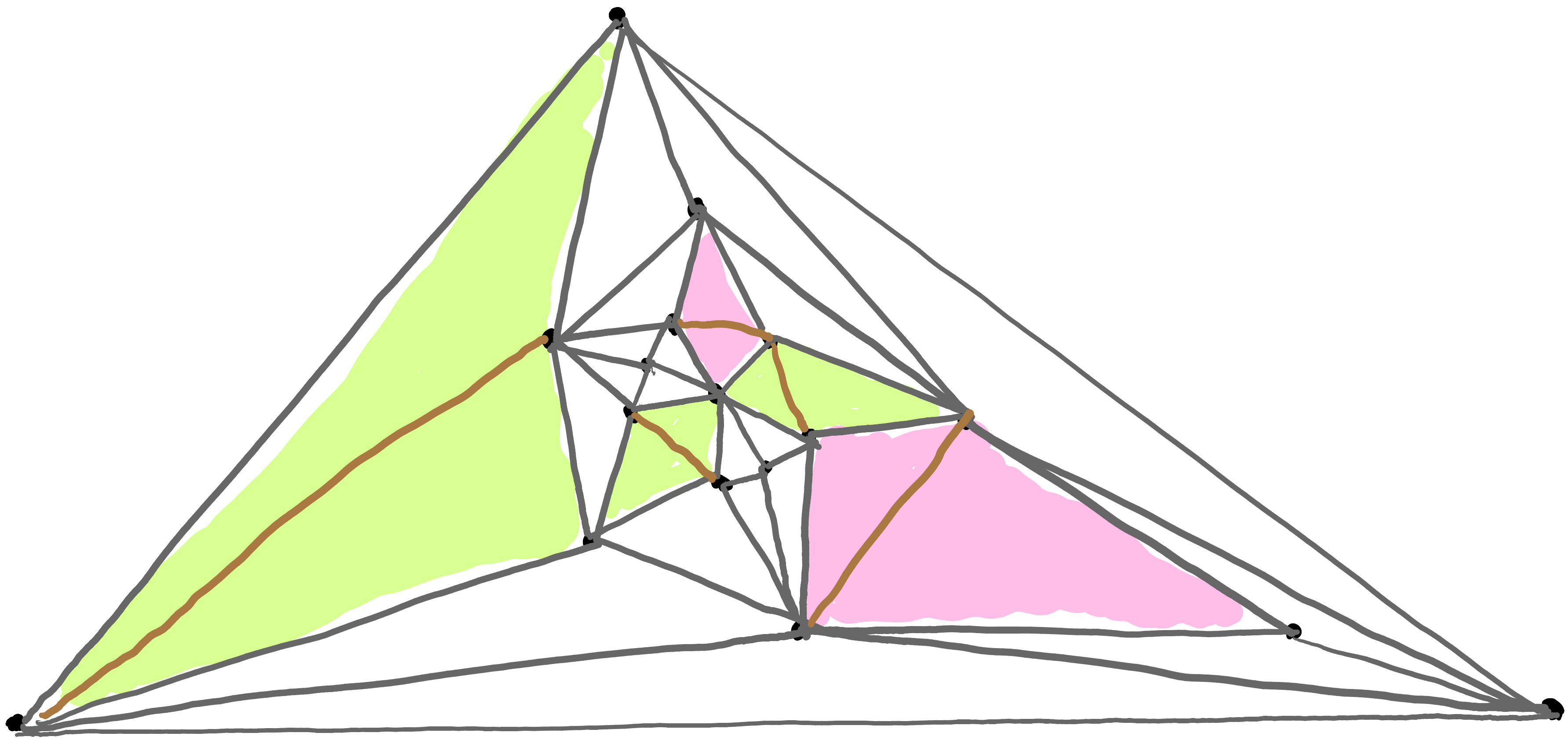
refinement

Level 0









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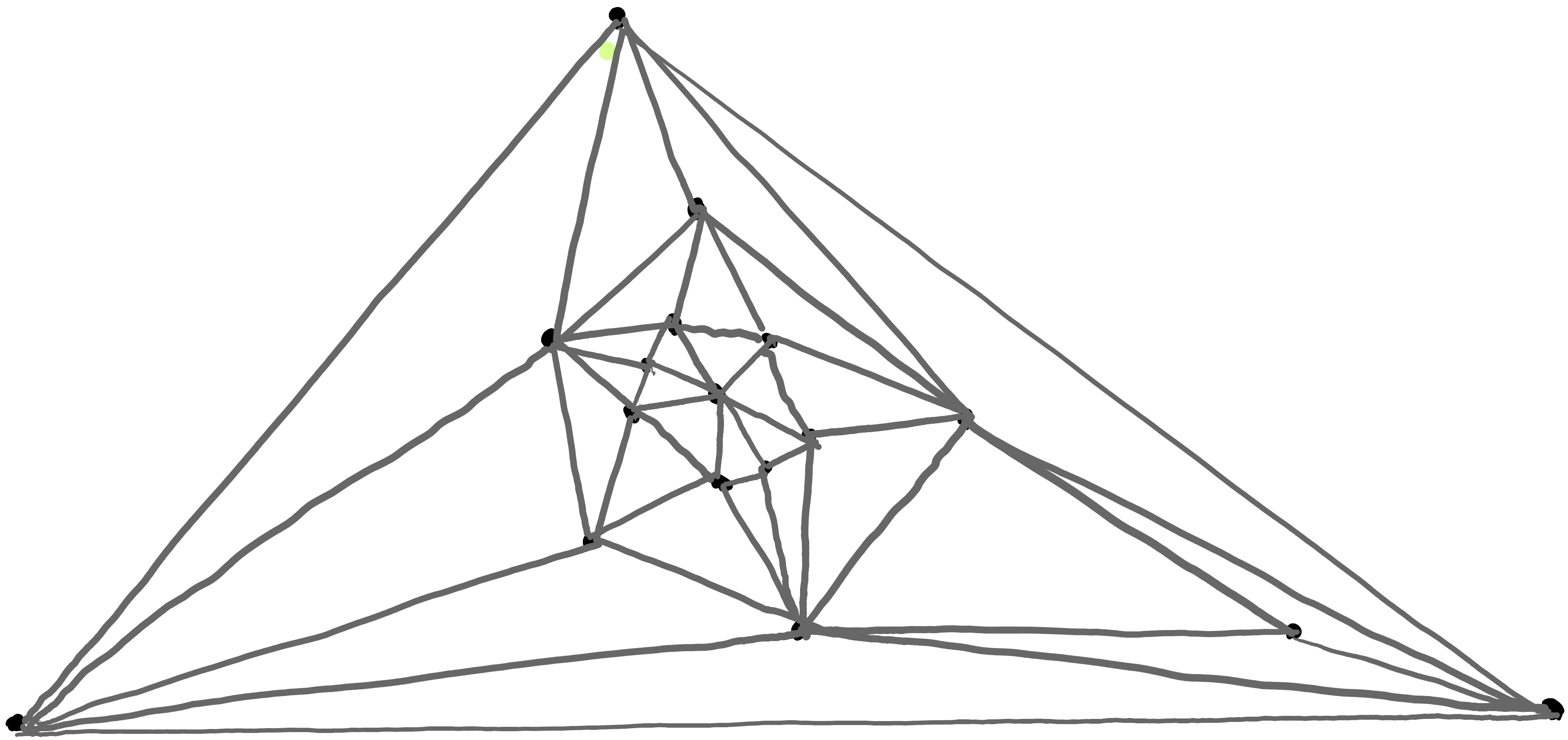

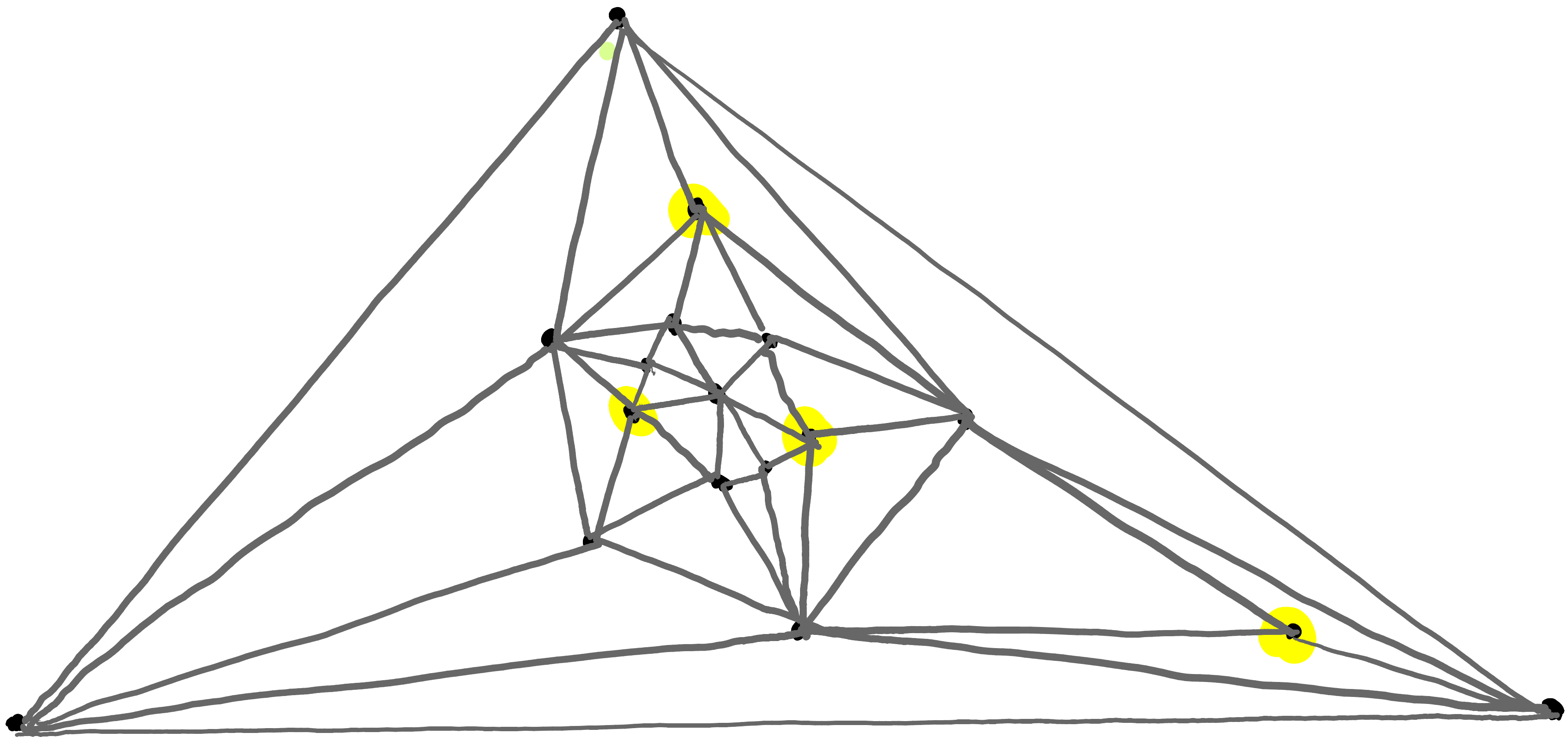

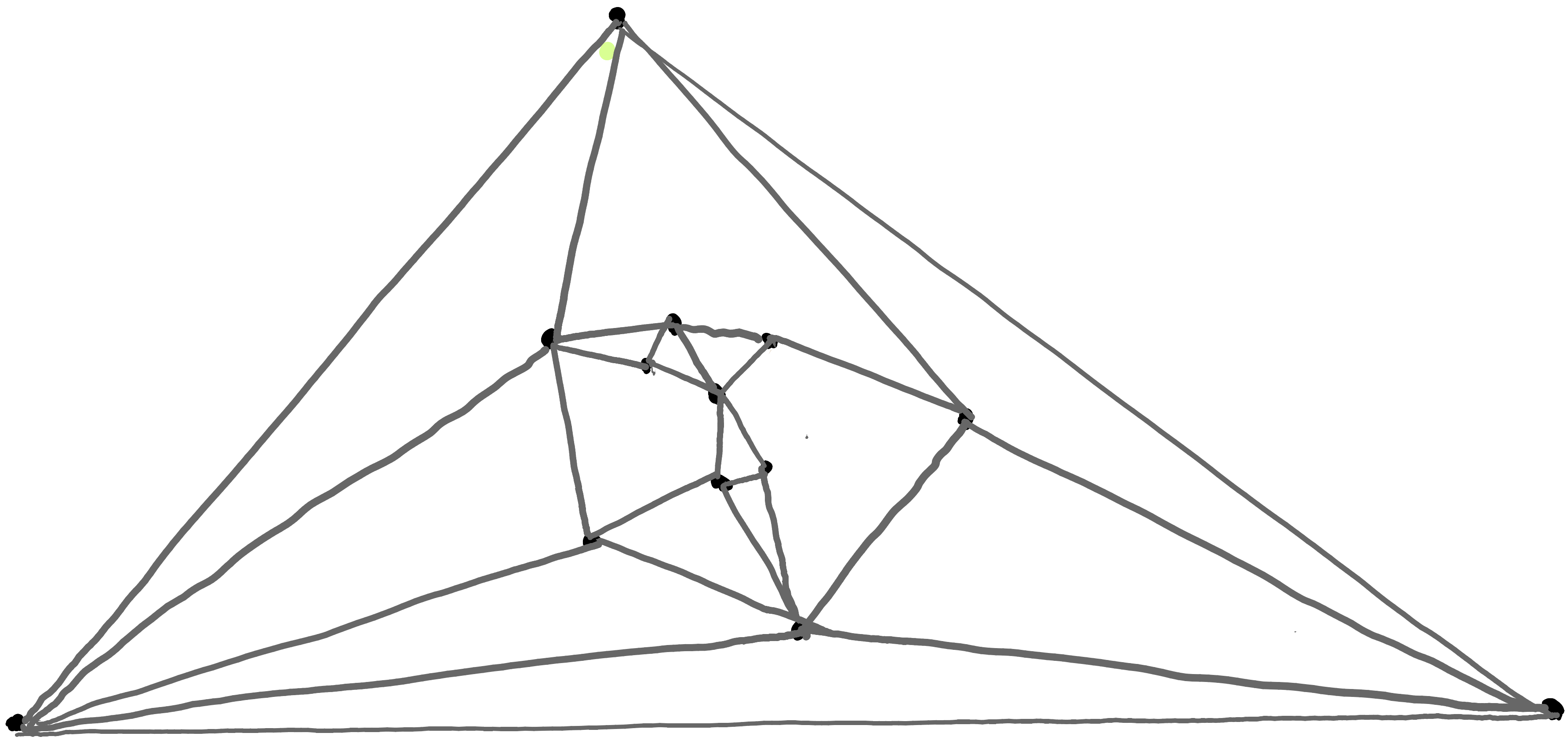

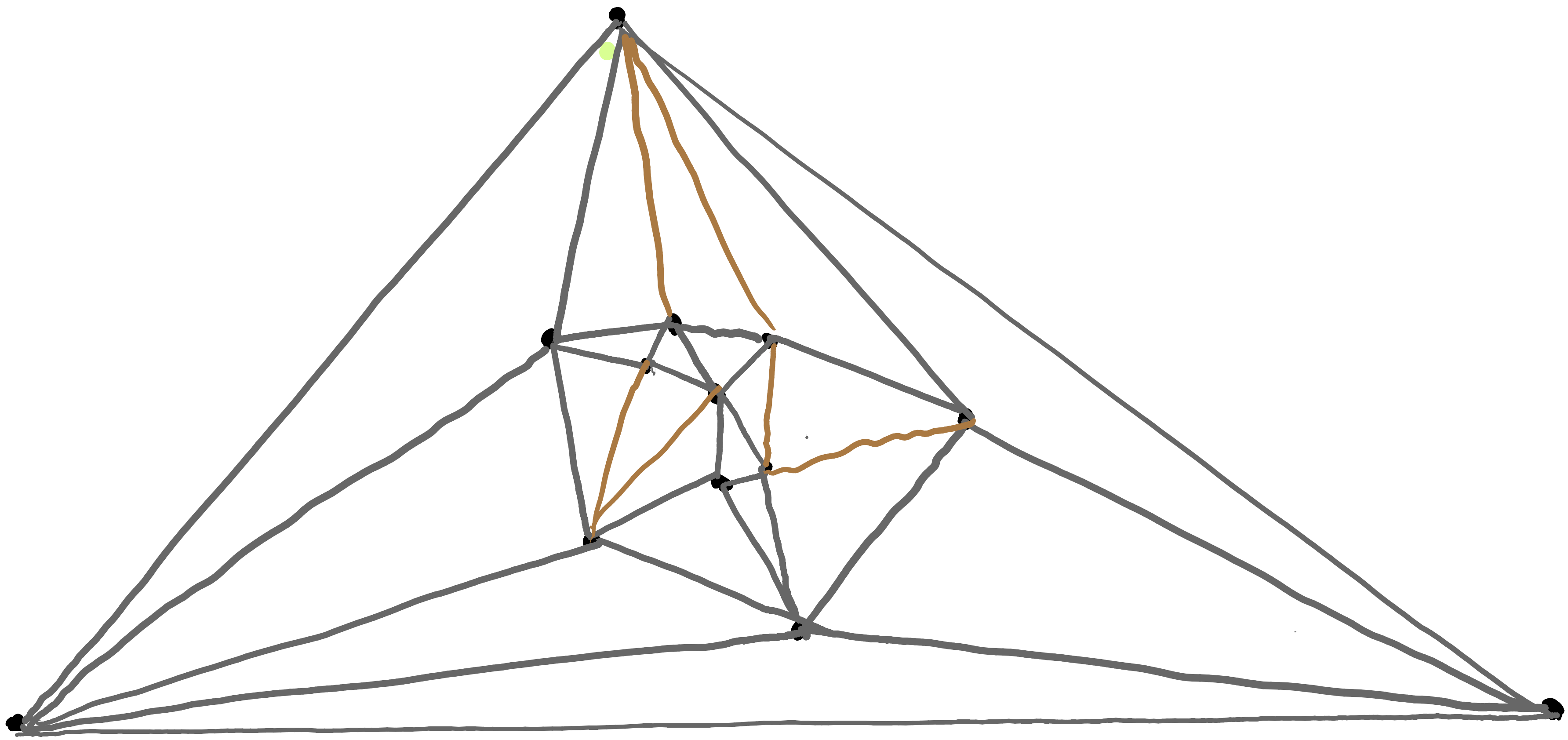

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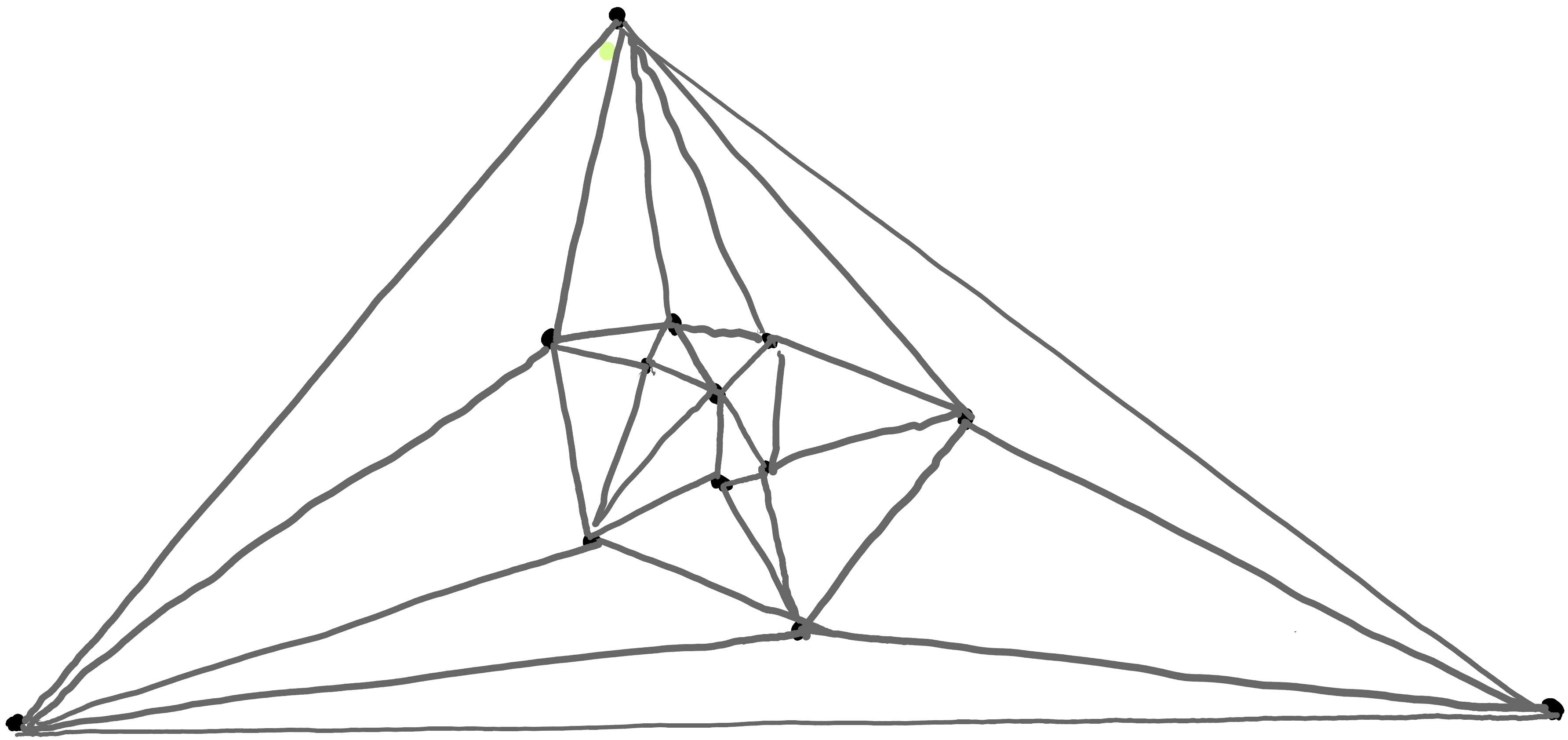
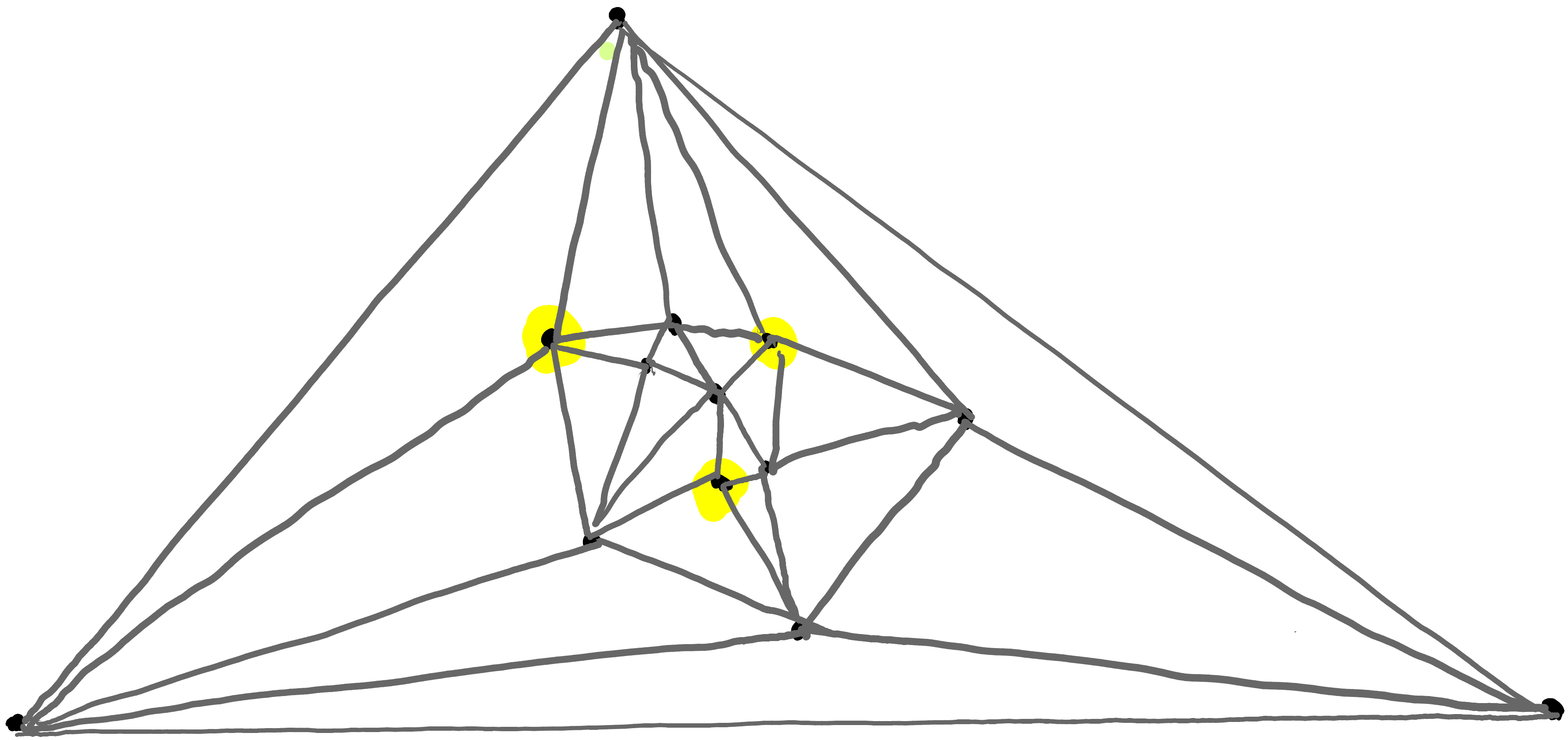
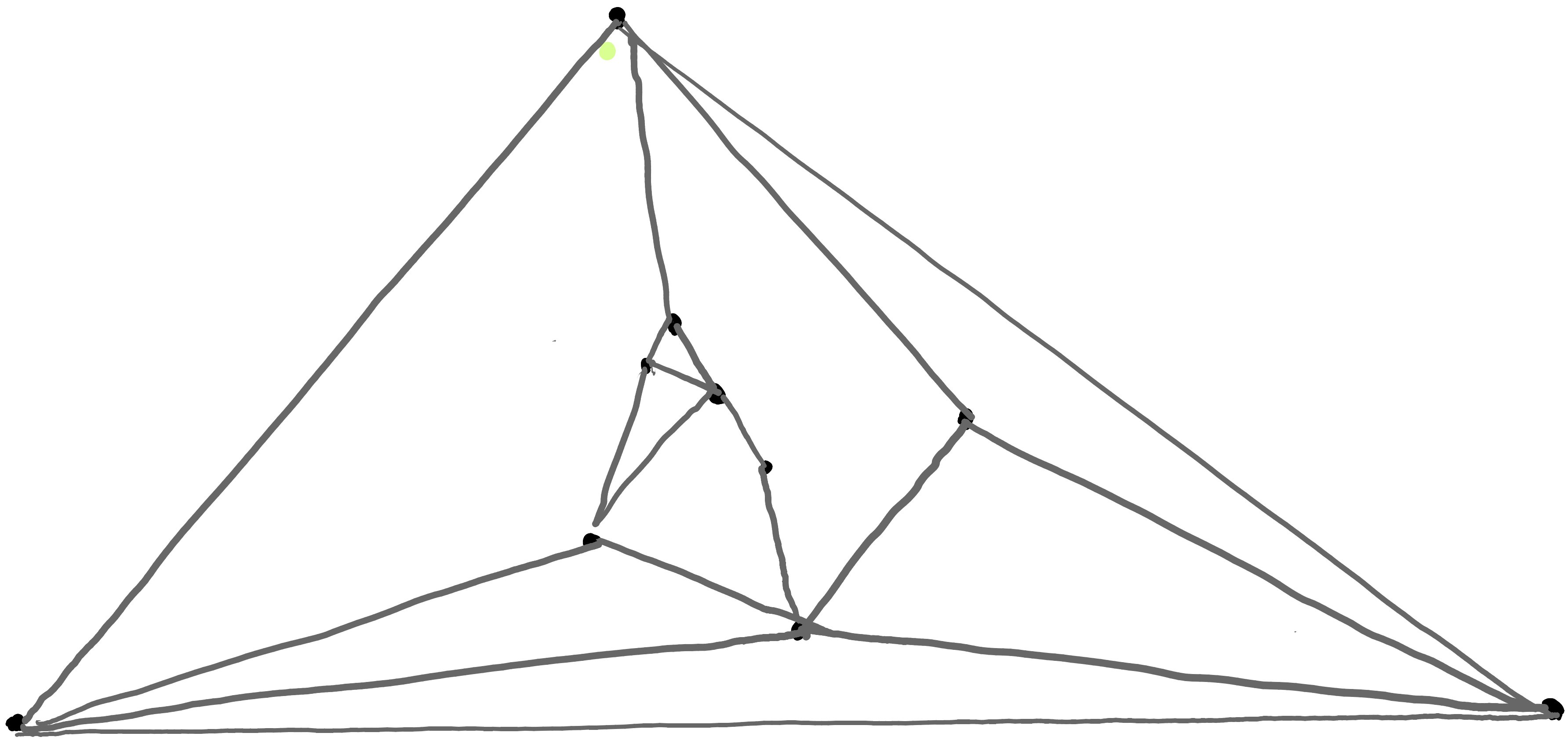
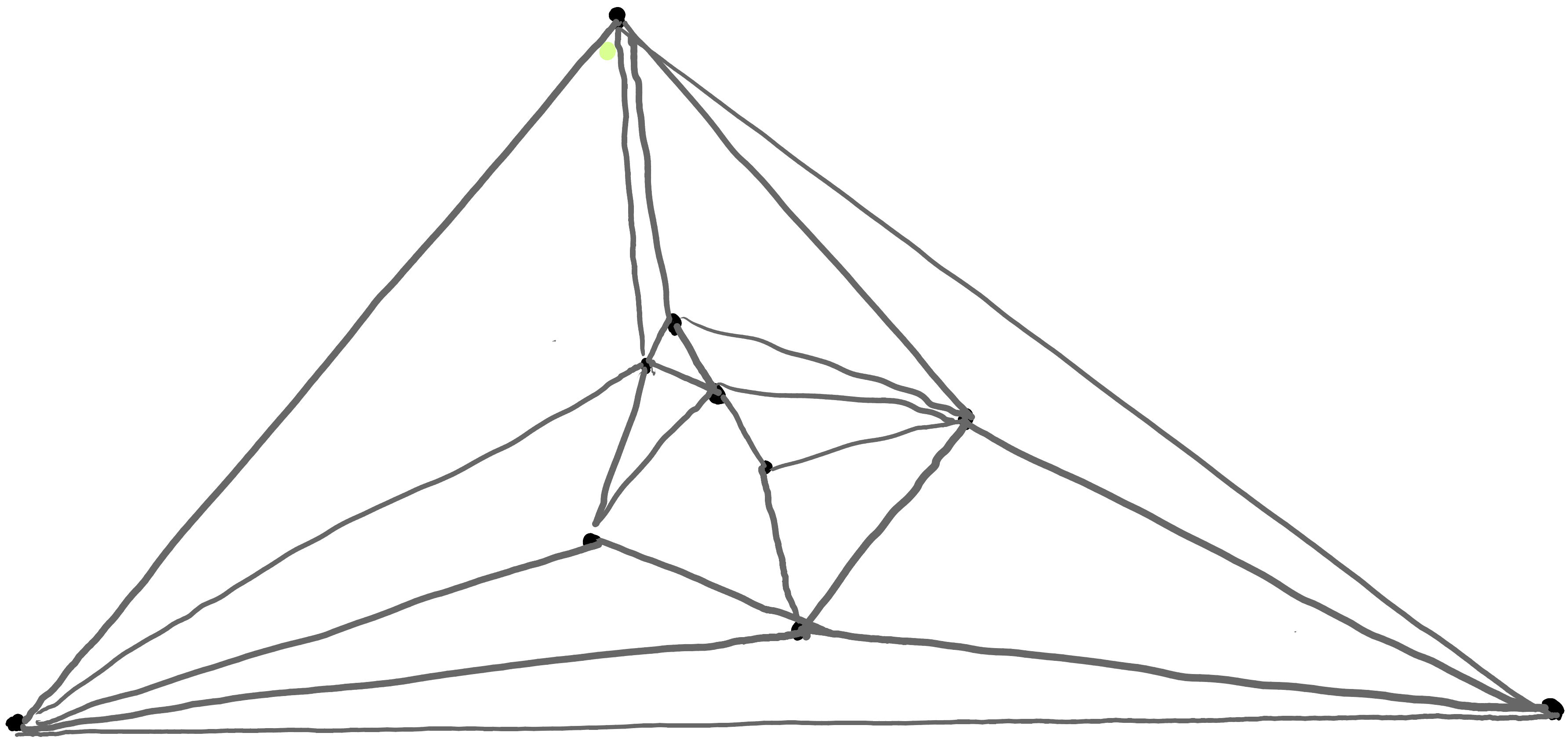
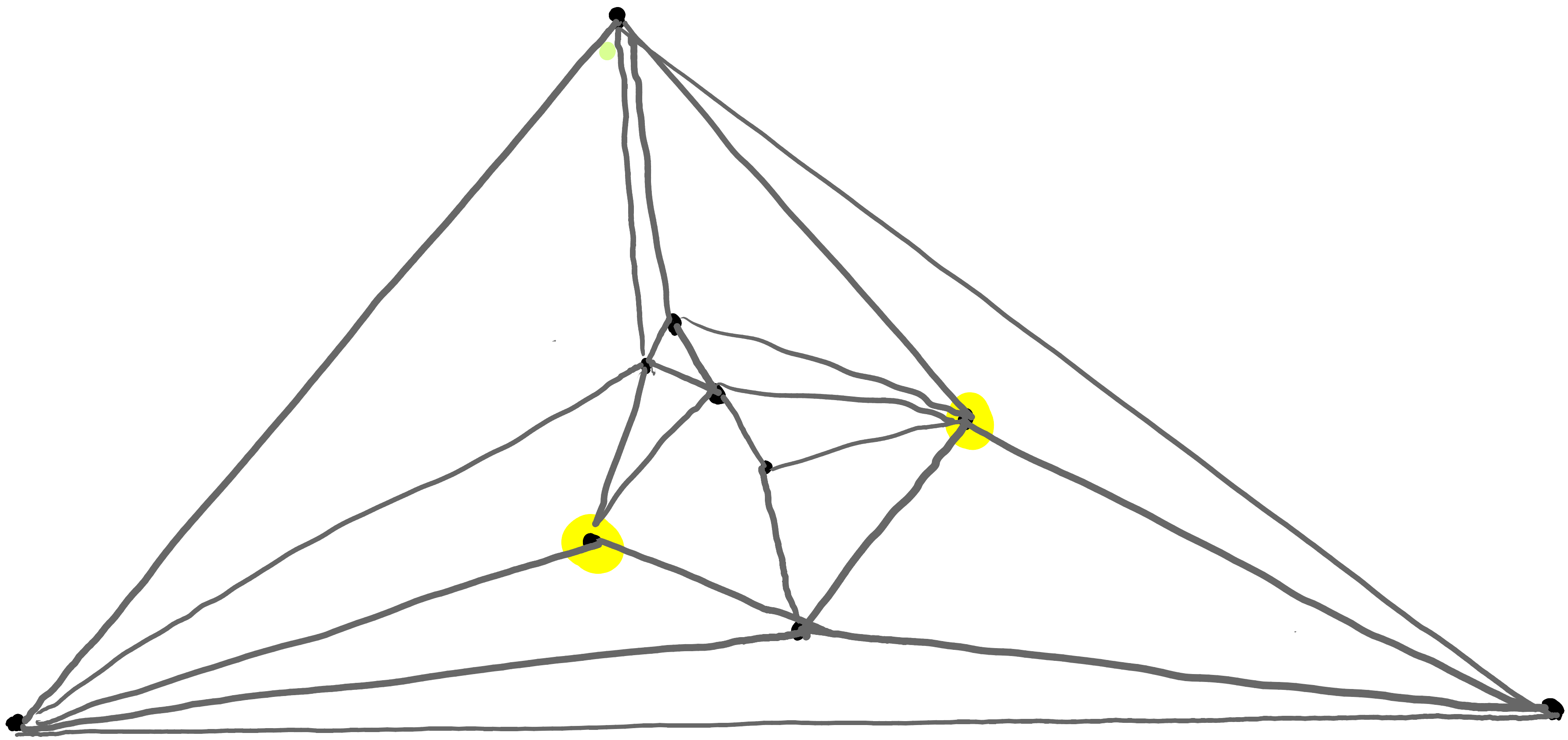


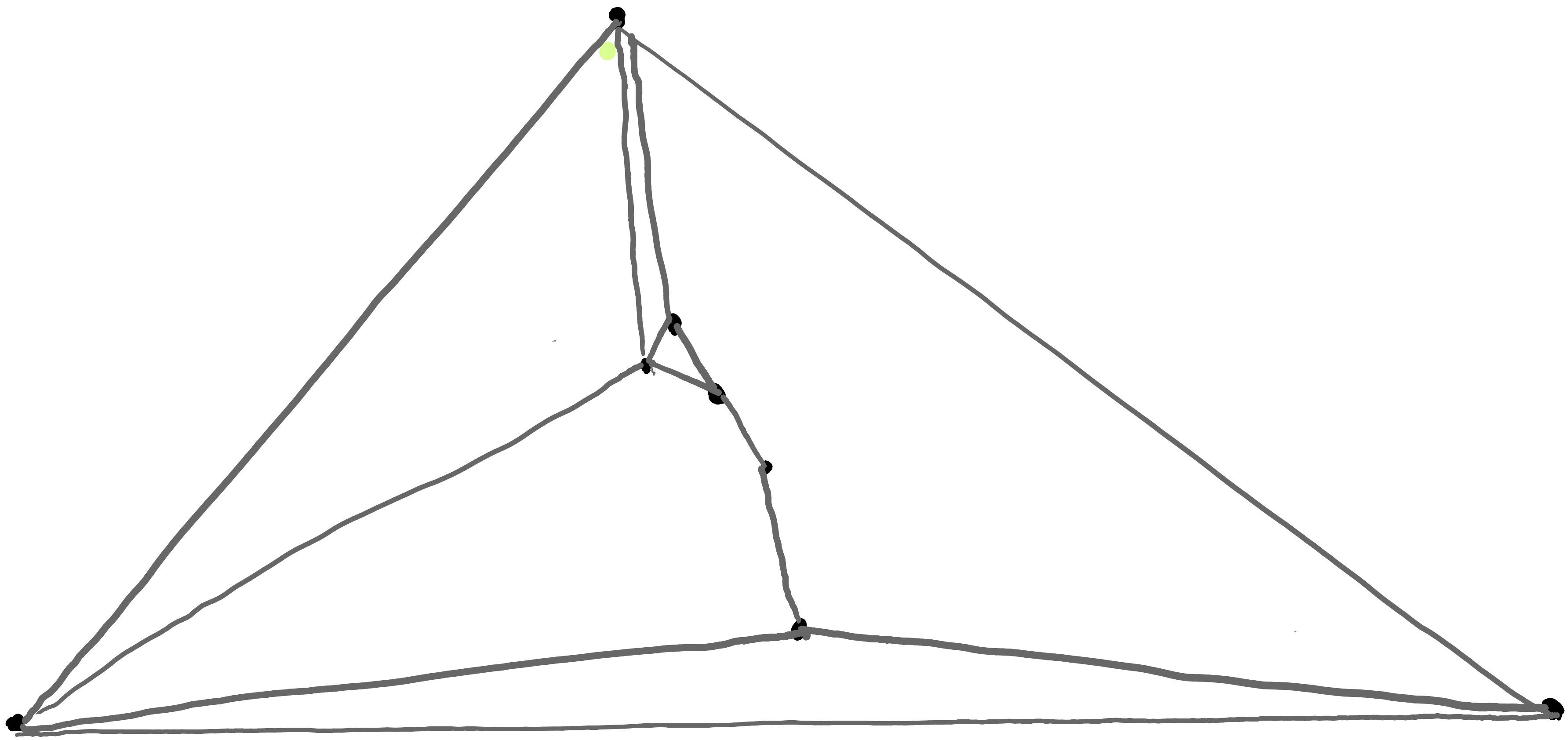




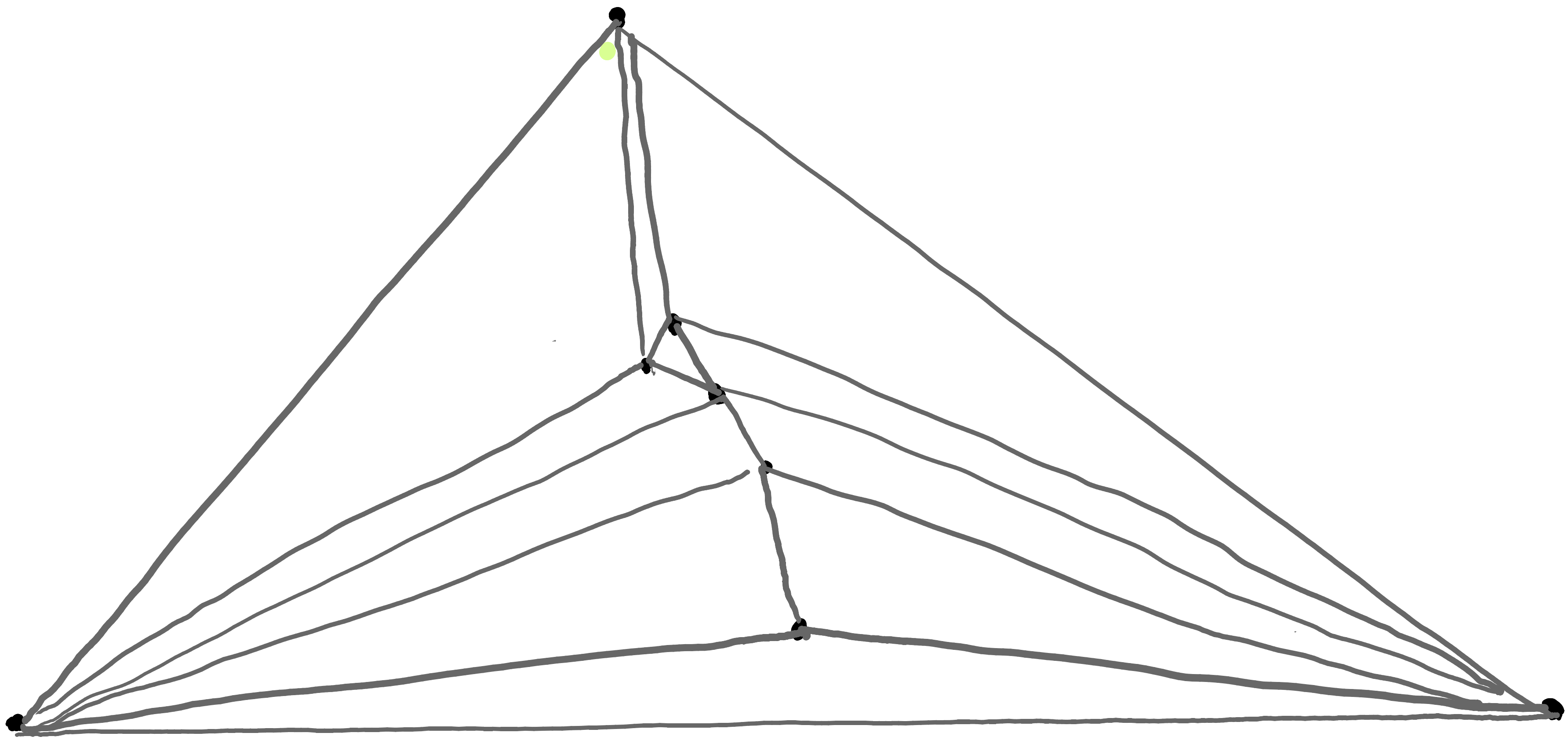
Level 3

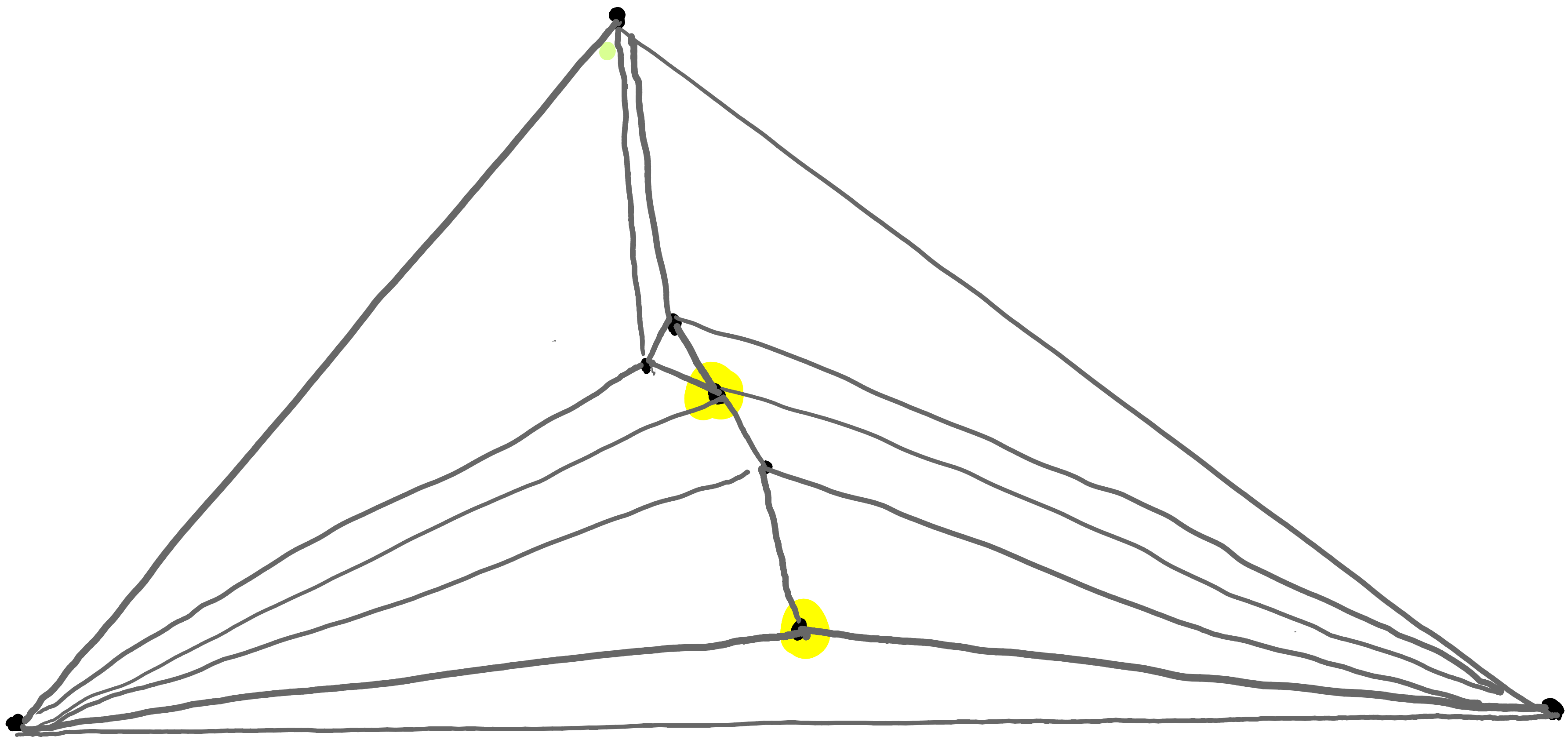


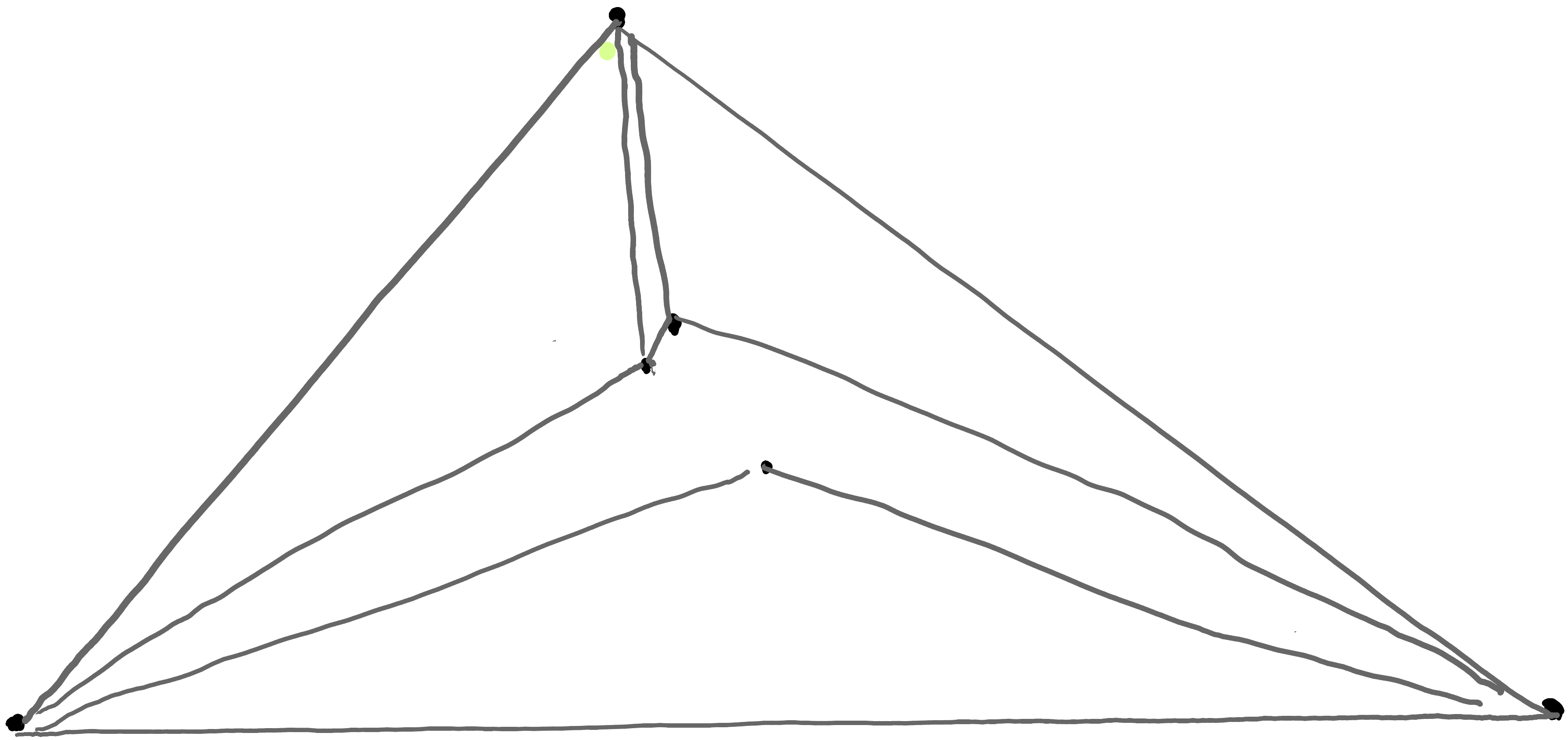




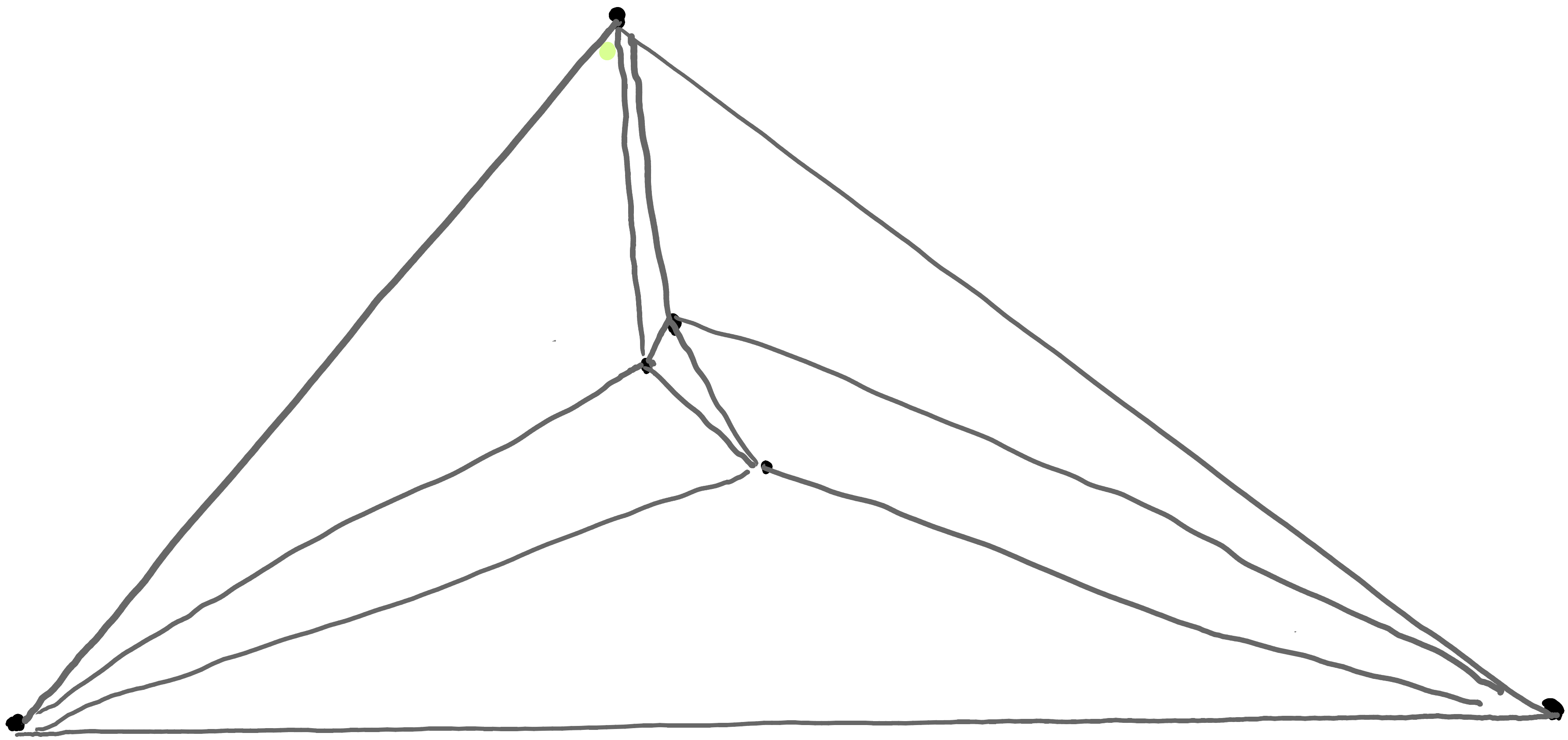
Level 4

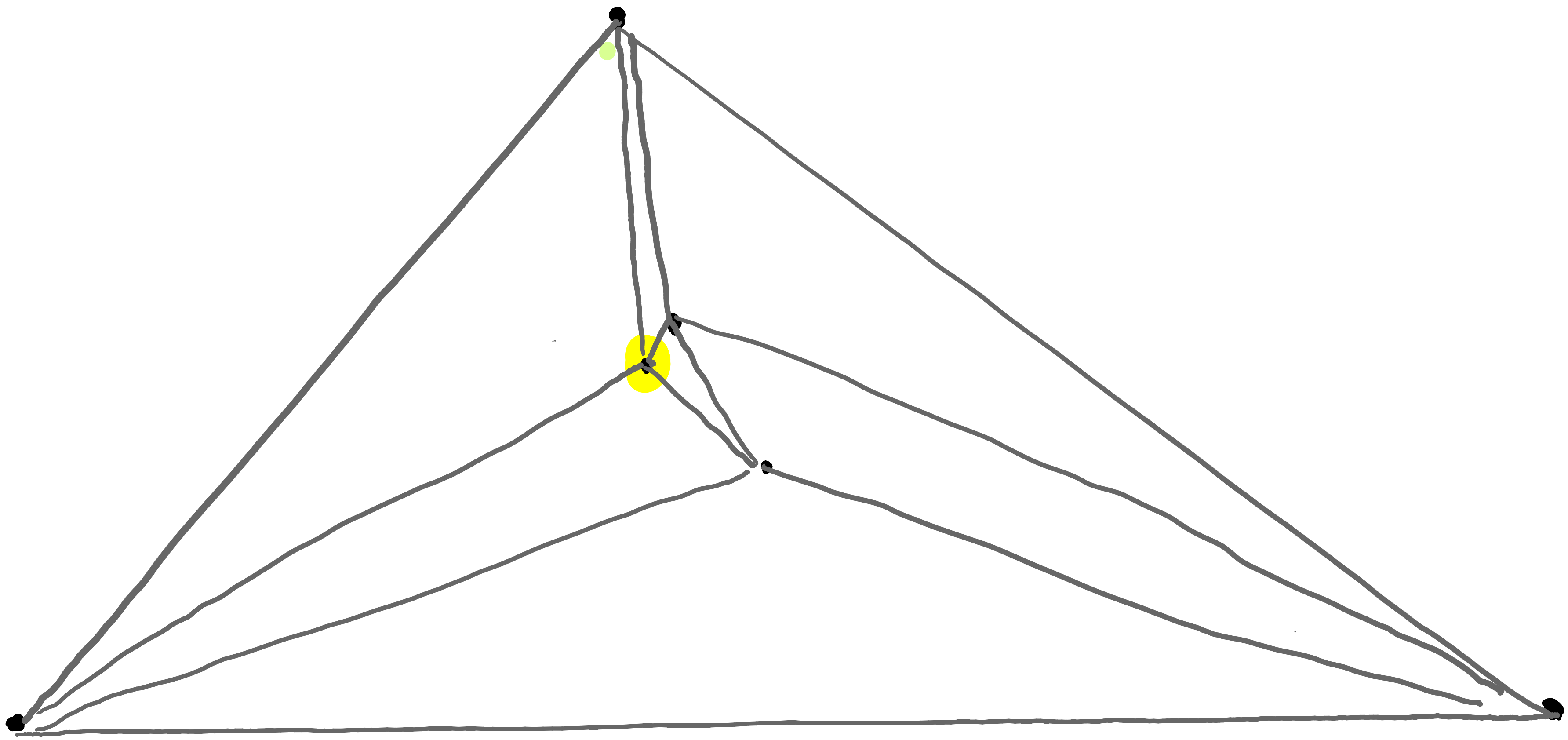


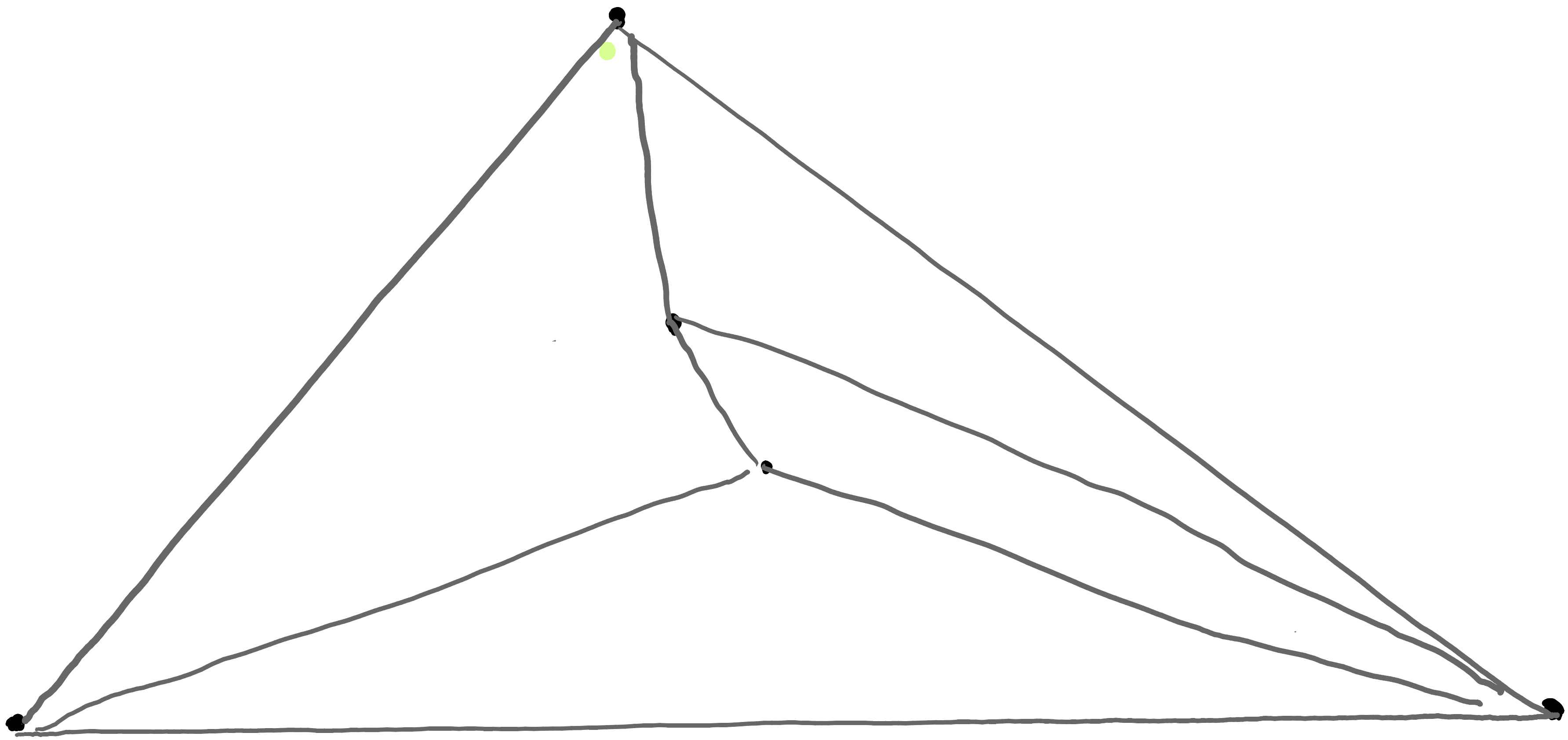




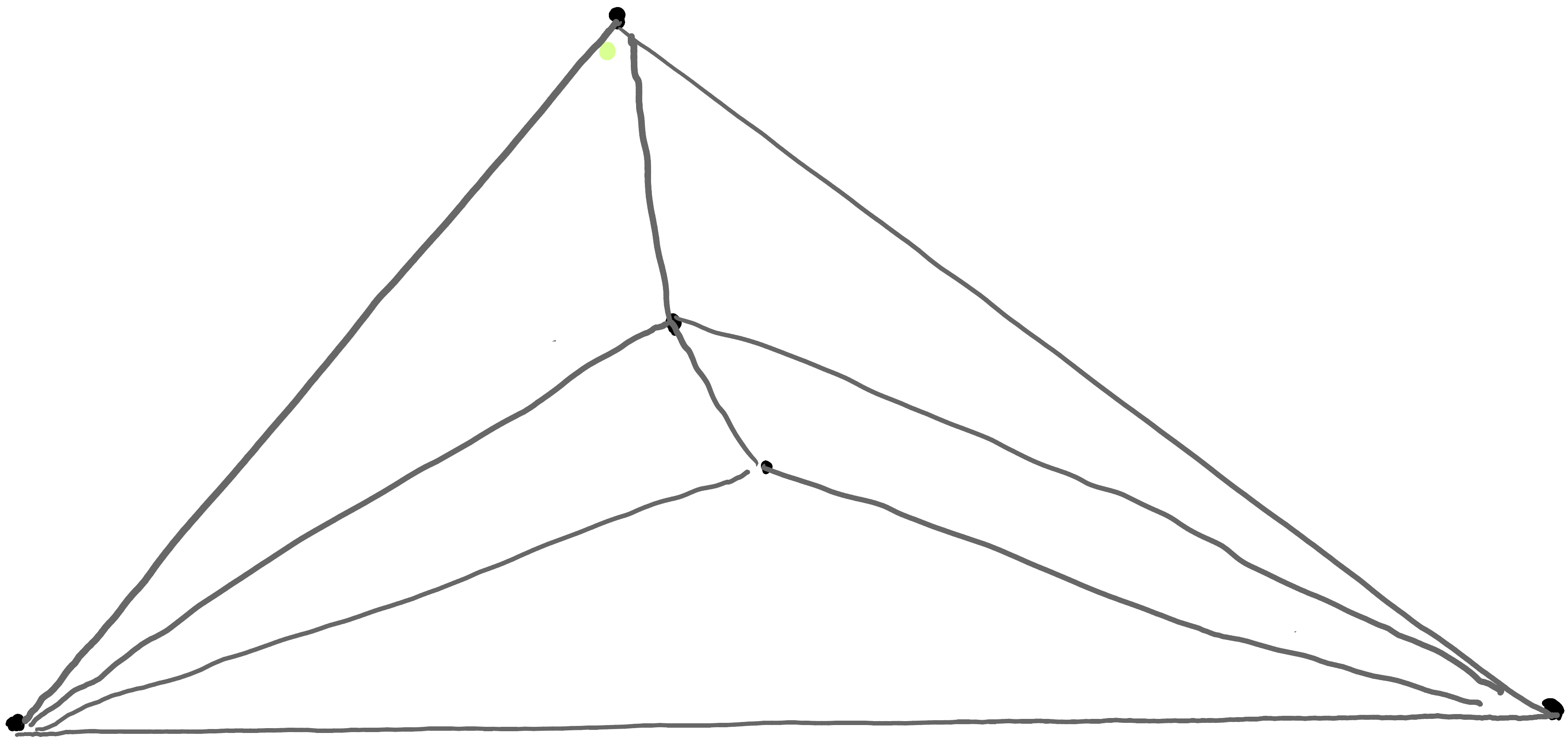
Level 5

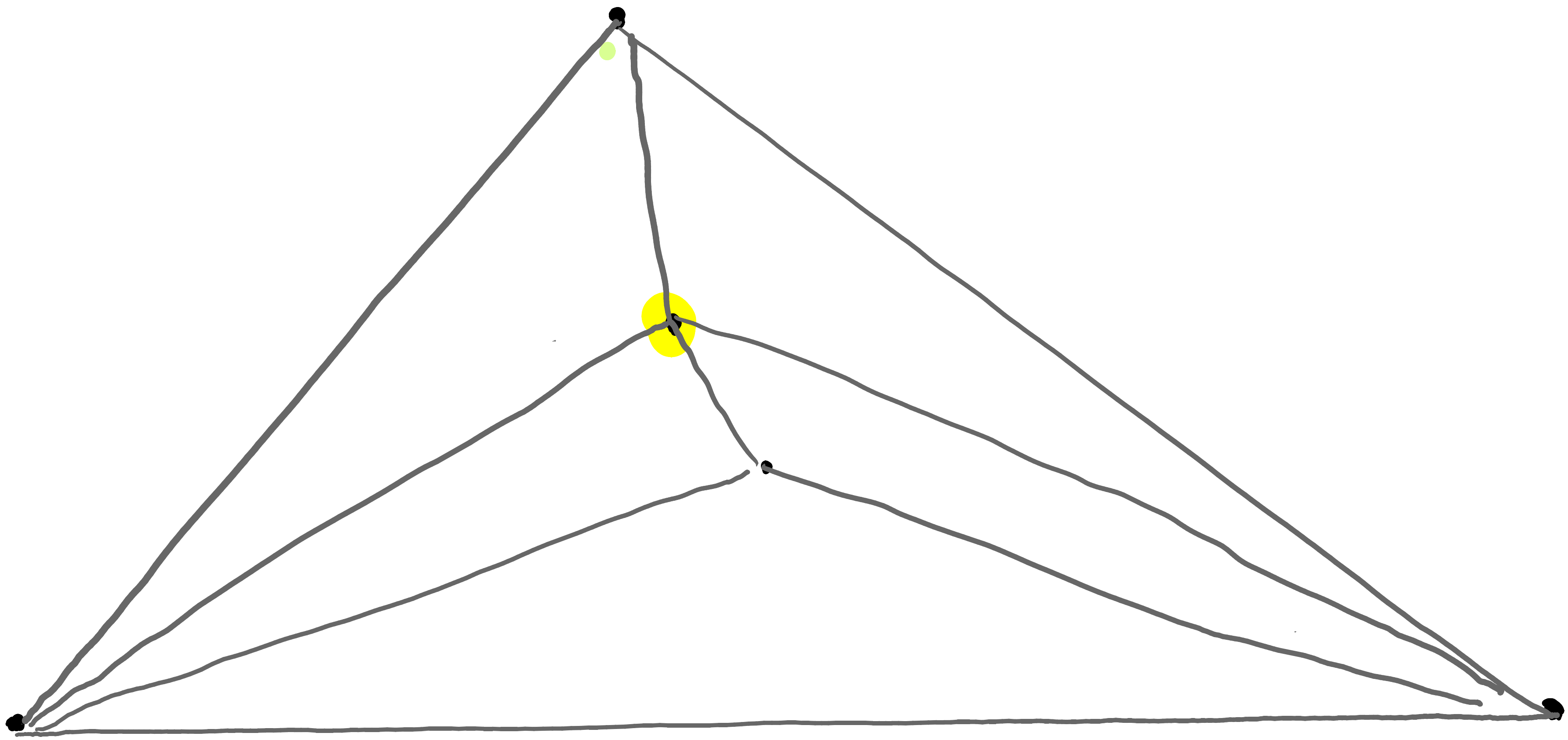


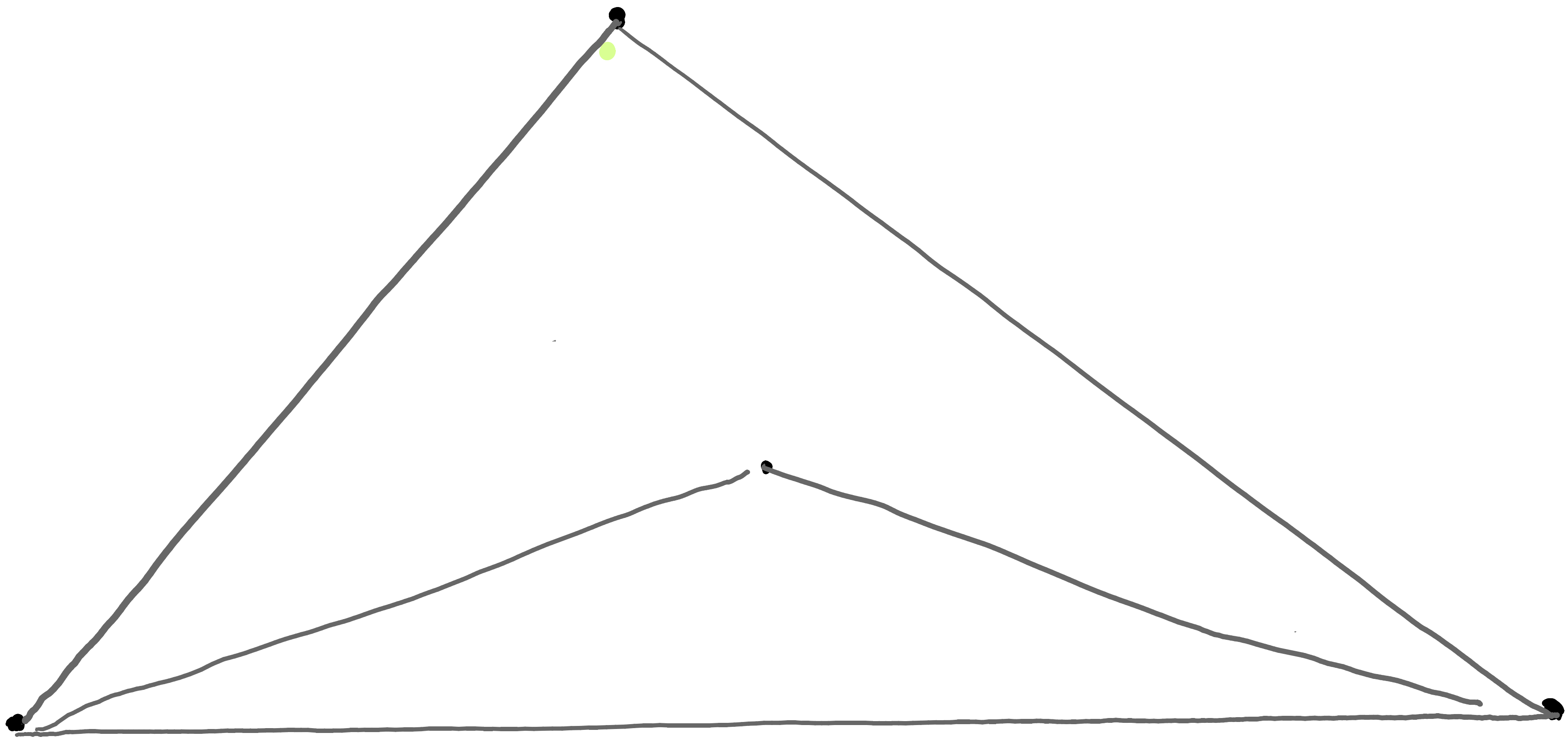




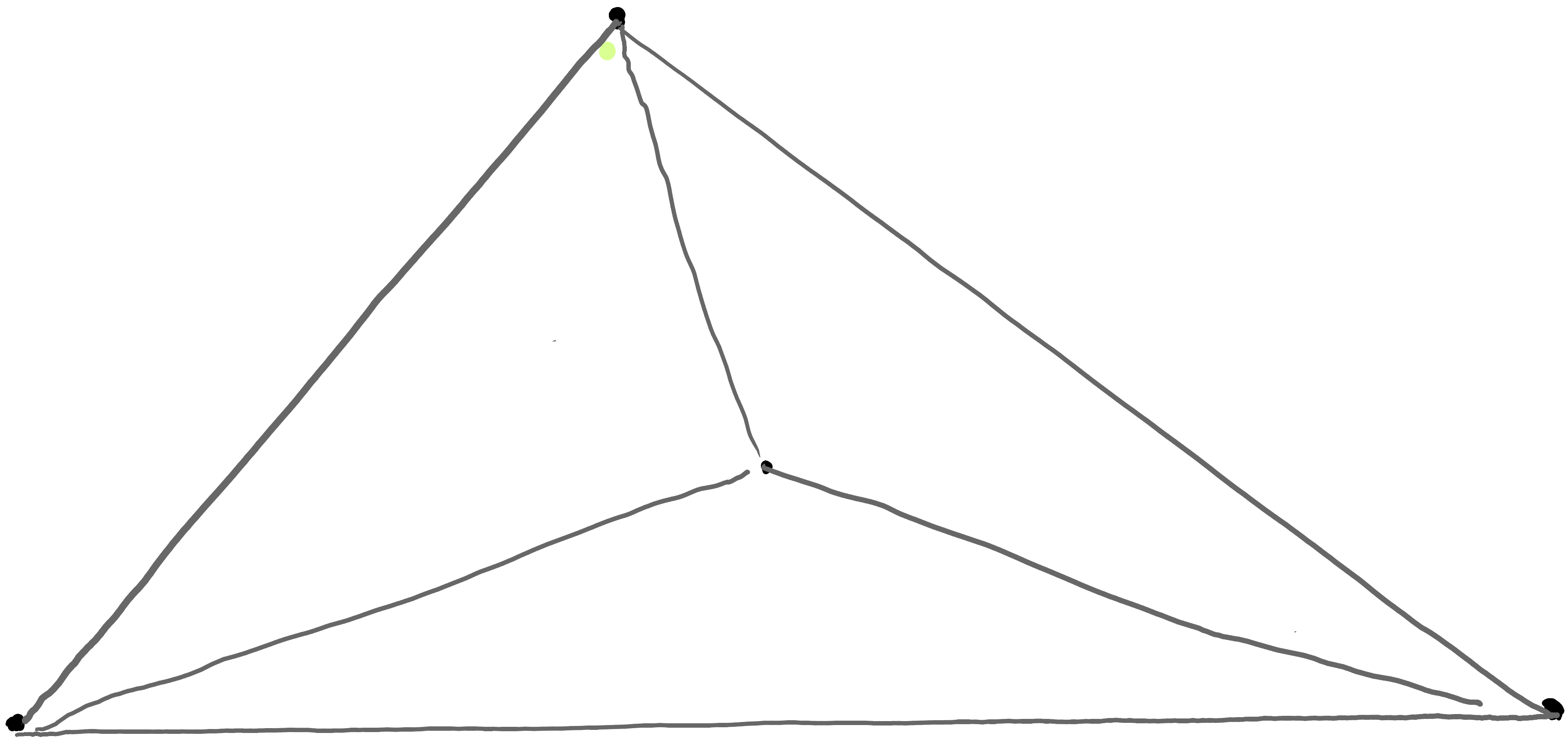
Level 6

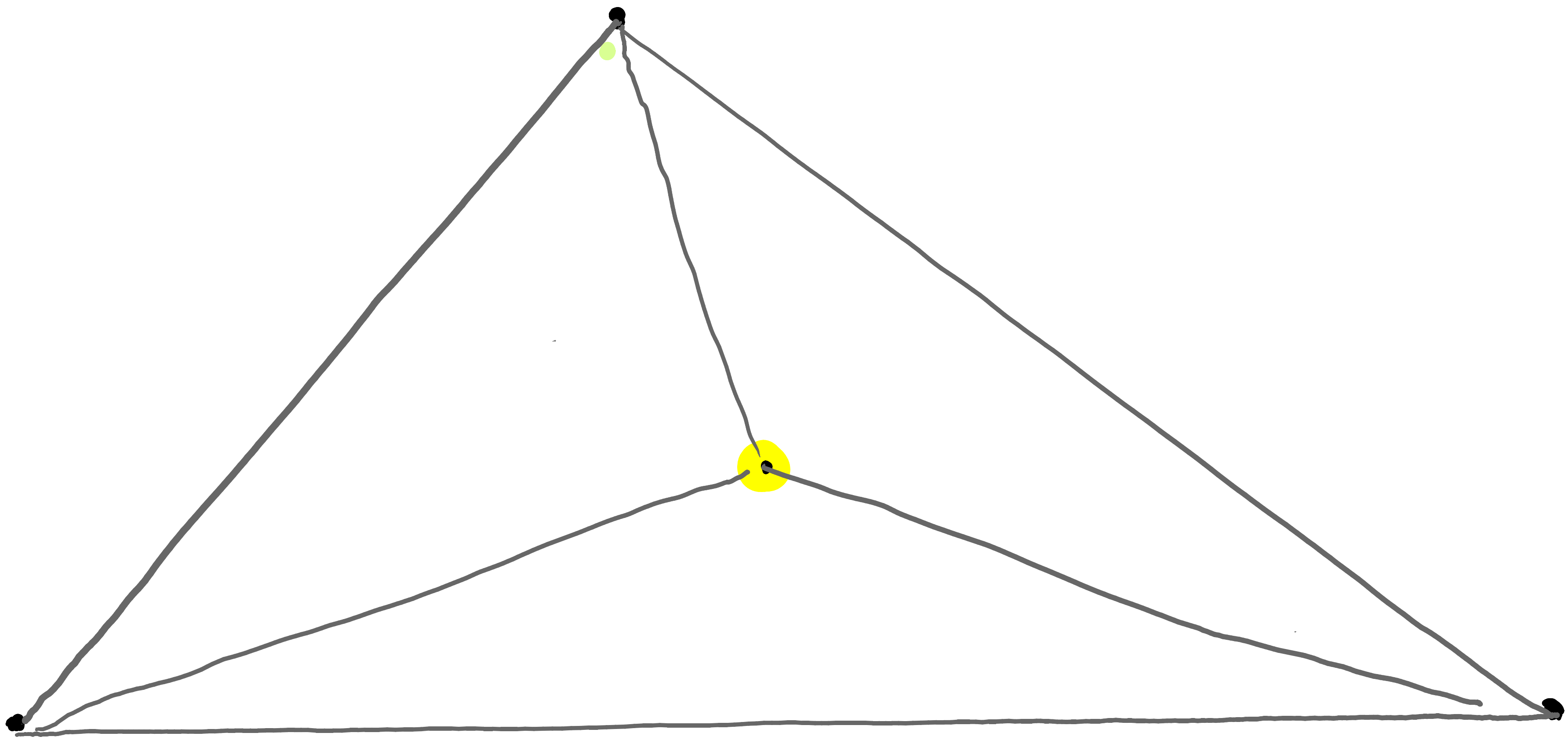




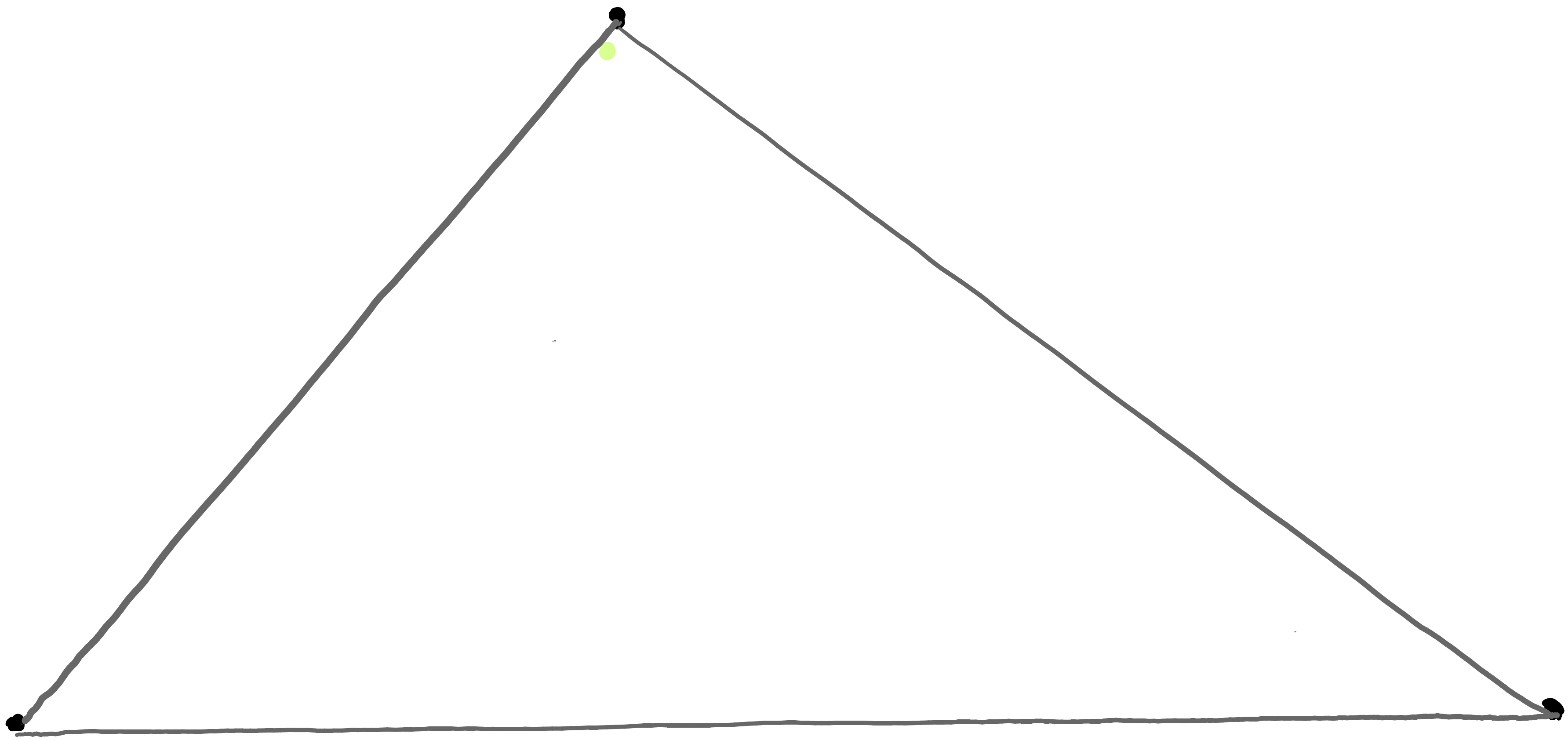


Level 7





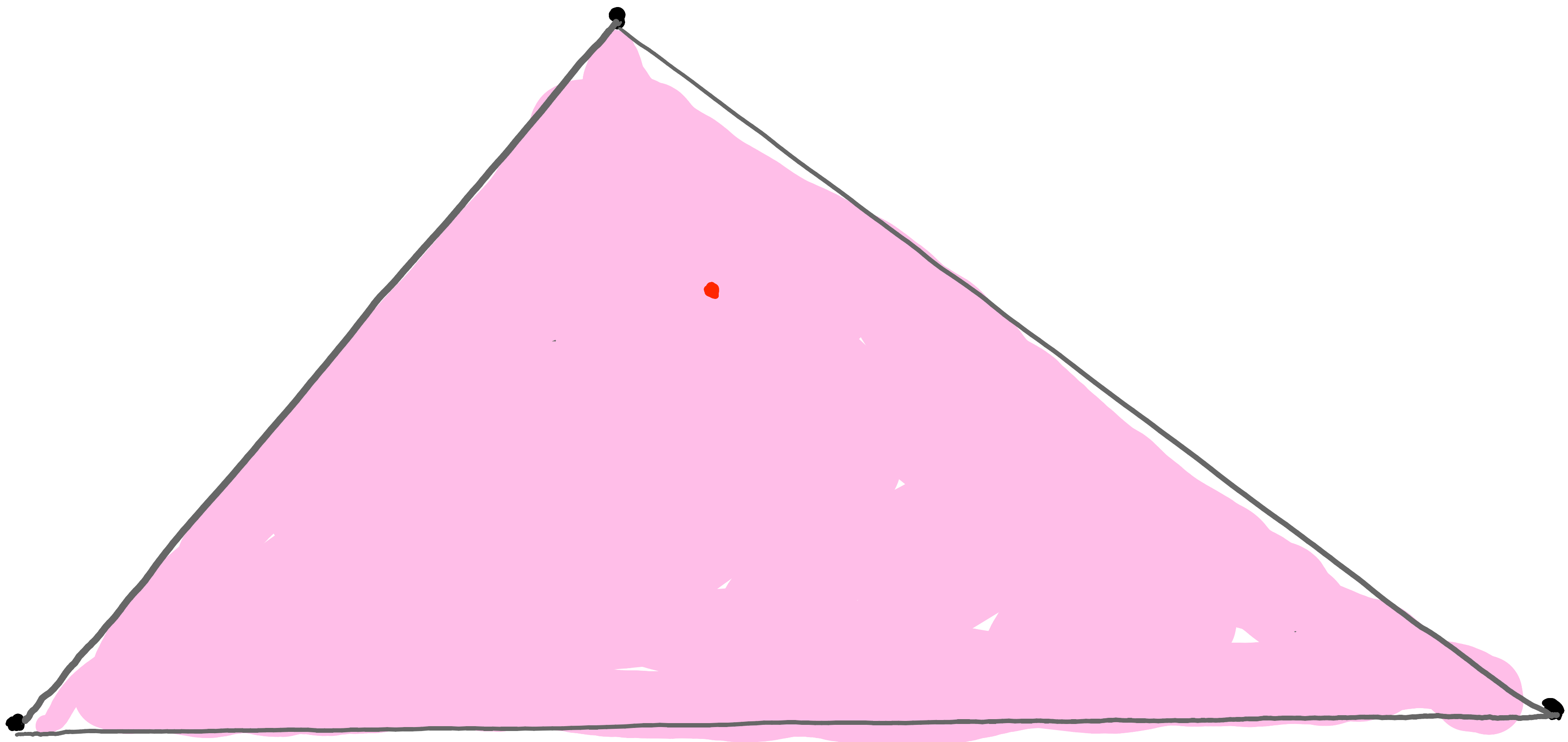
Level 8



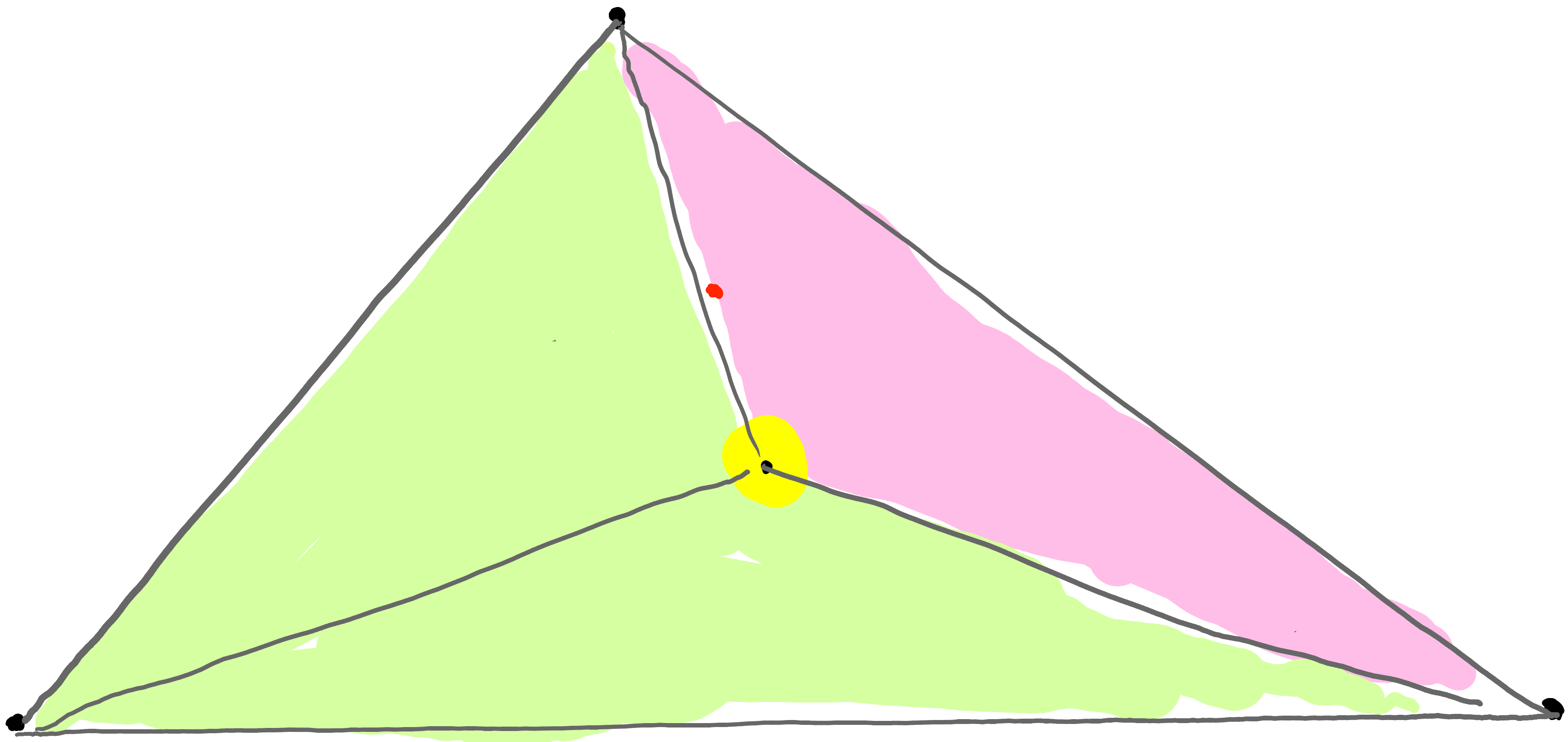
How many levels?

$$\log_{\frac{15}{16}} n \approx 10.7 \log_2 n$$

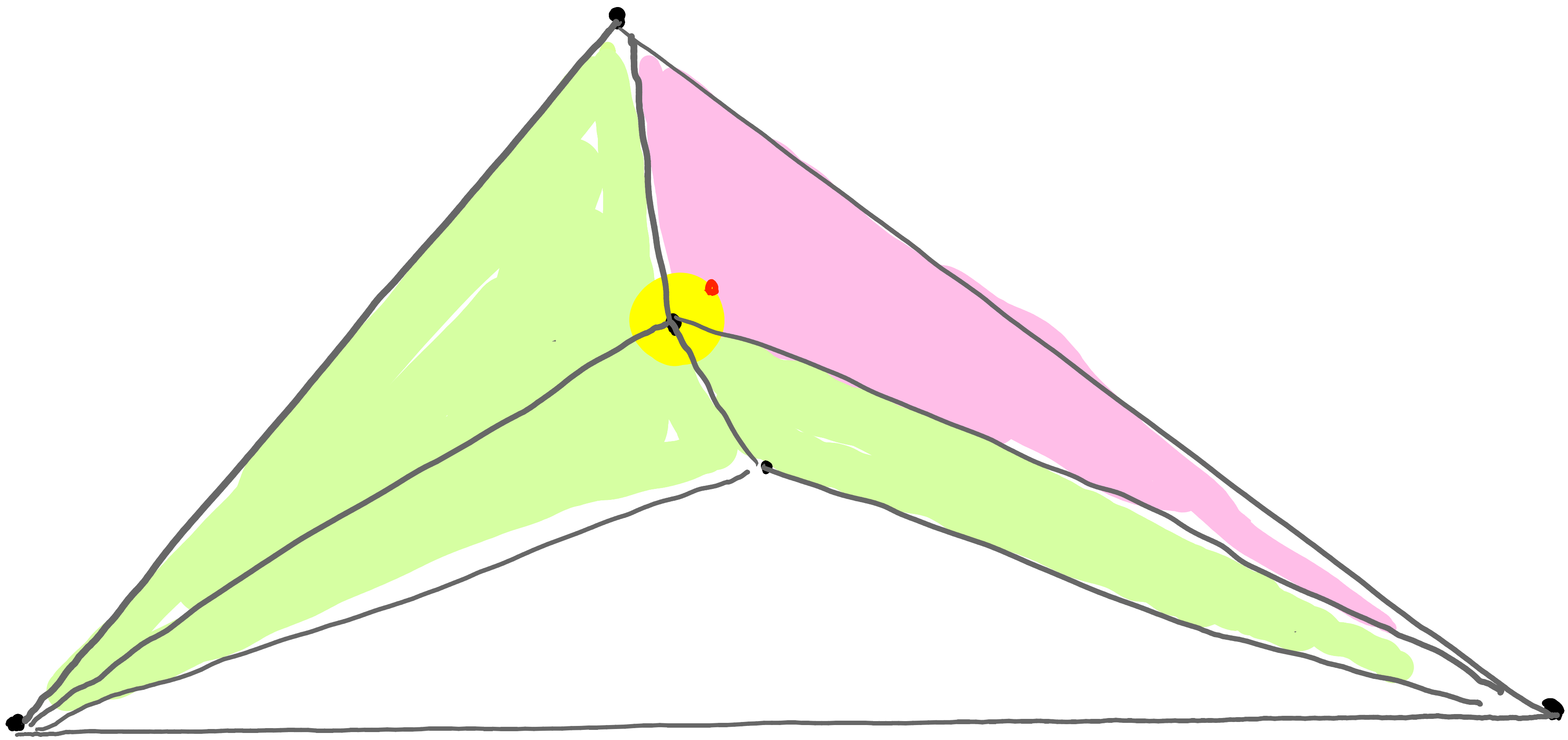
Level 8



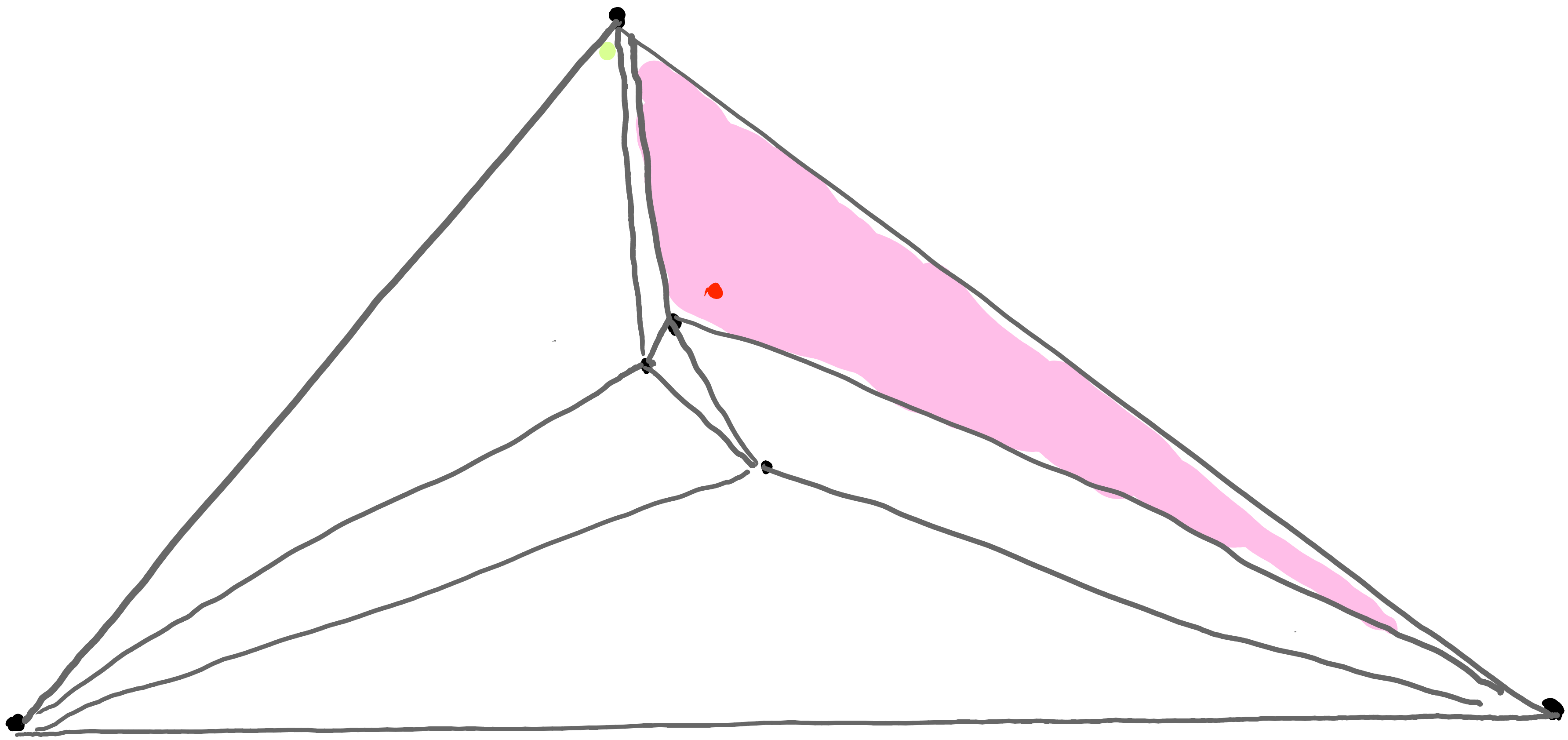
Level 7



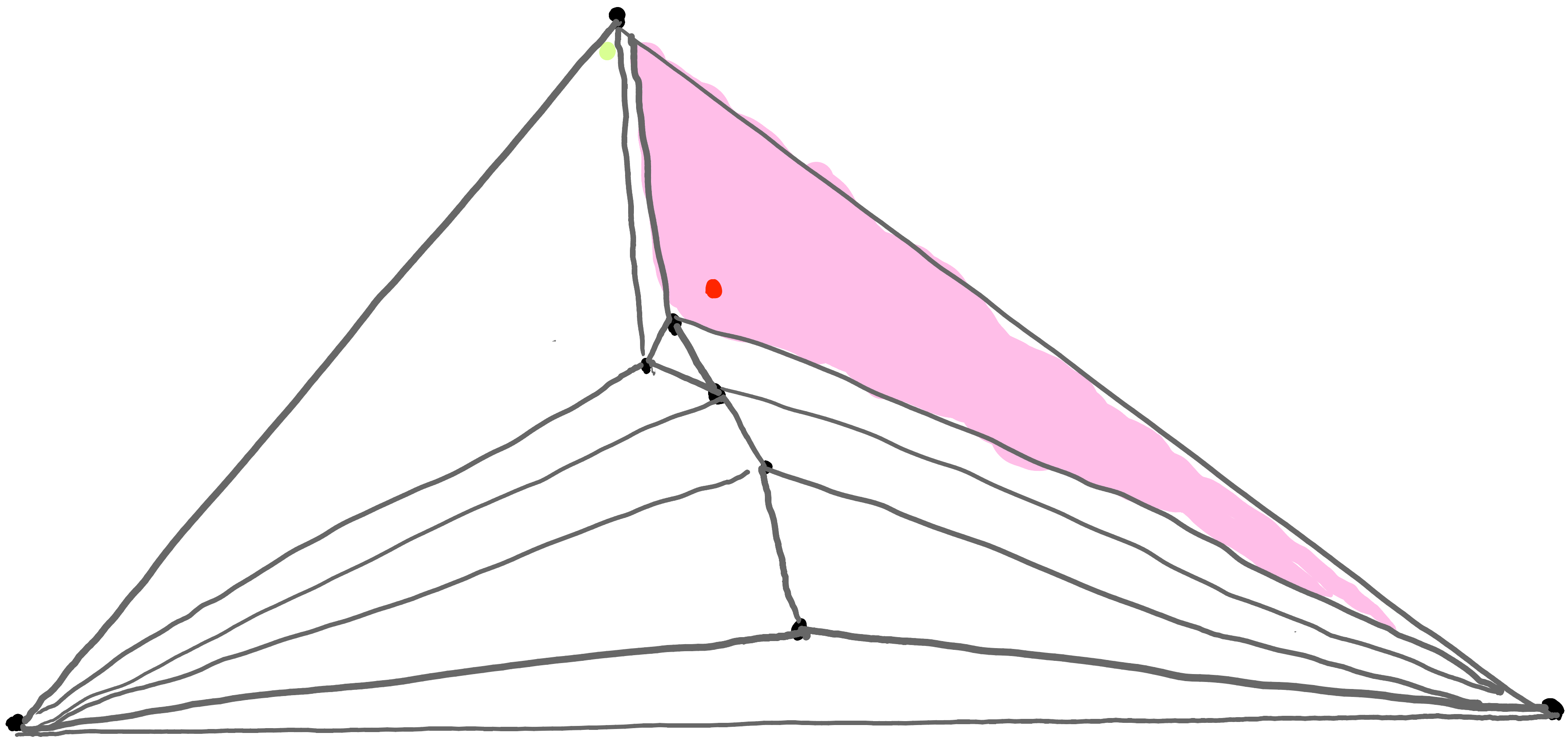
Level 6



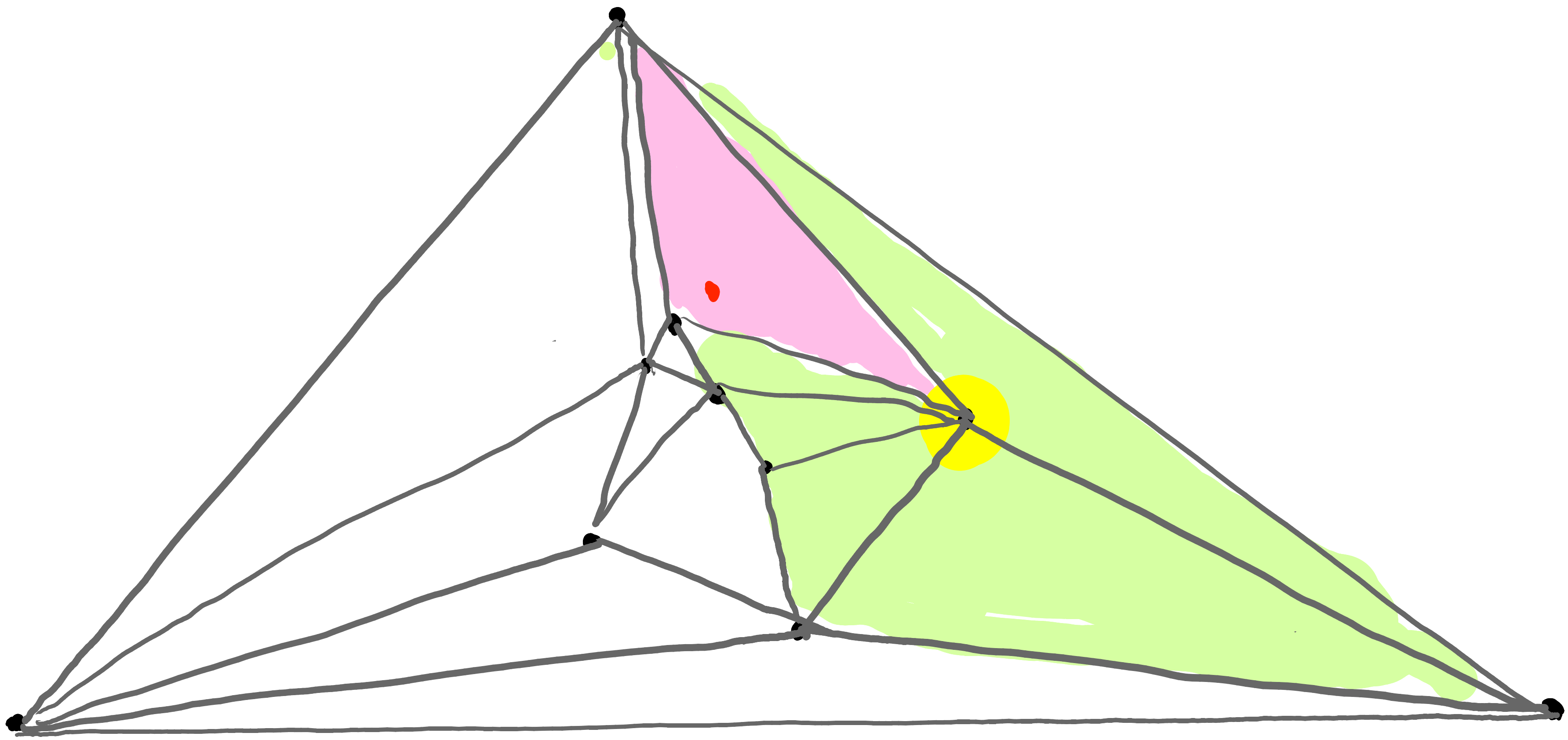
Level 5



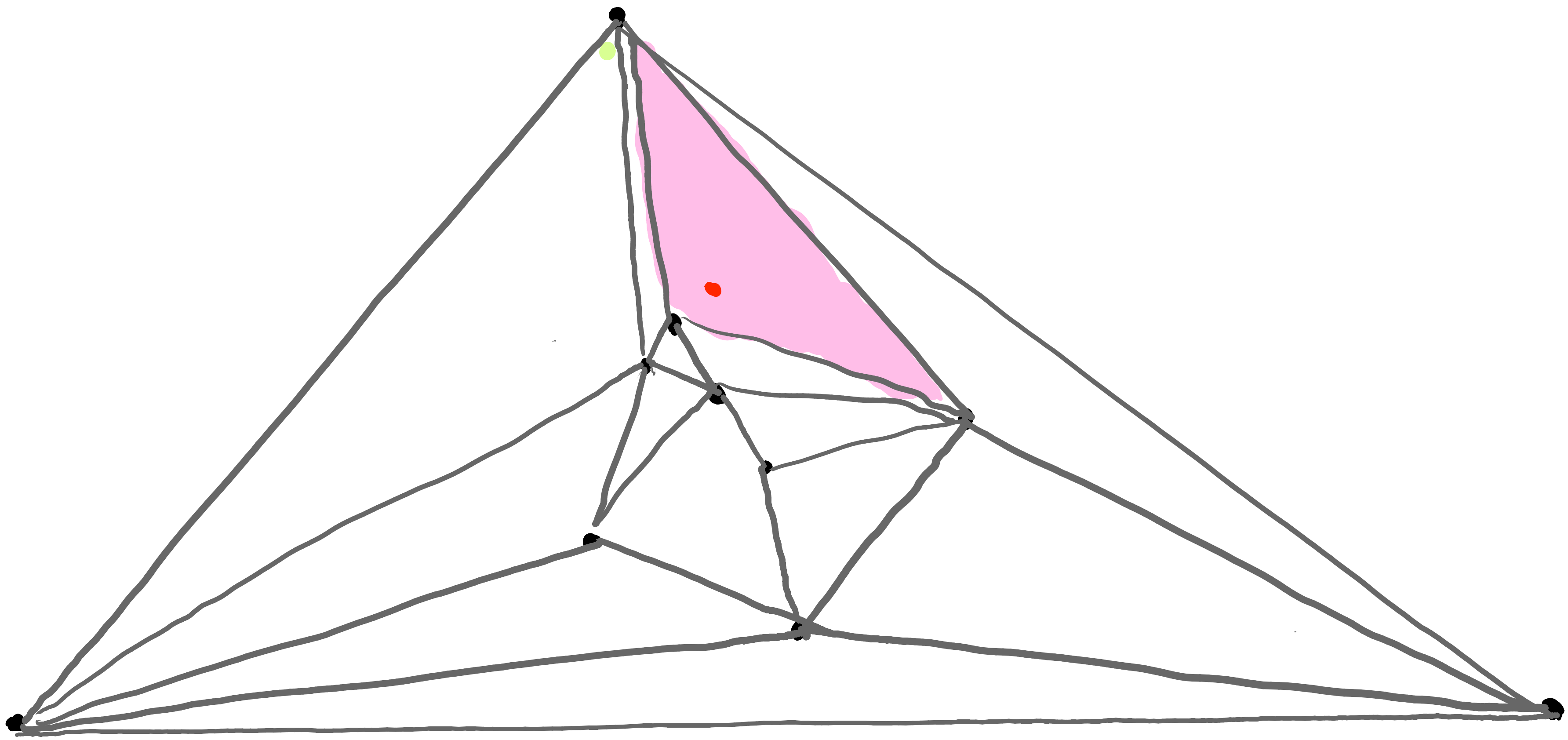
Level 4



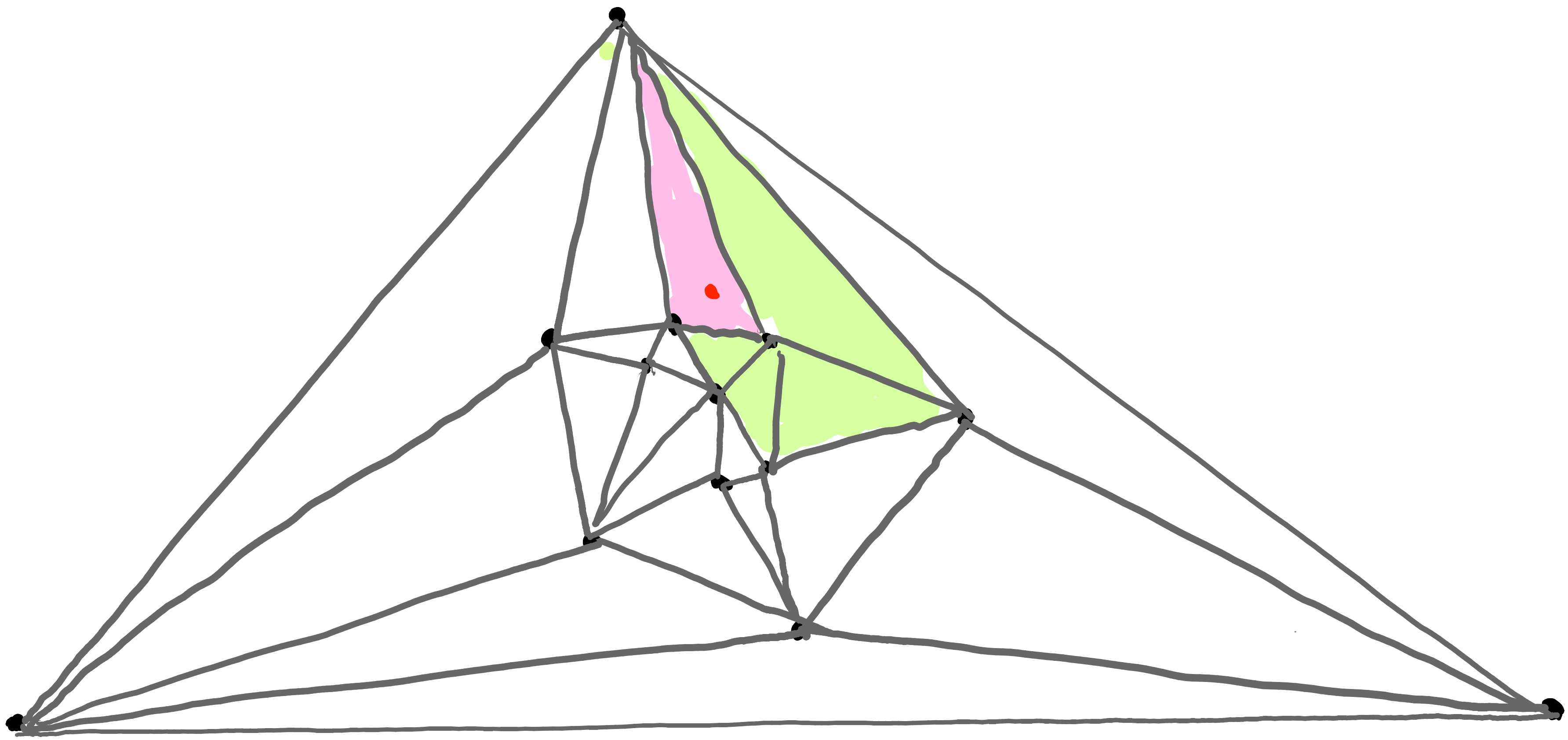
Level 3



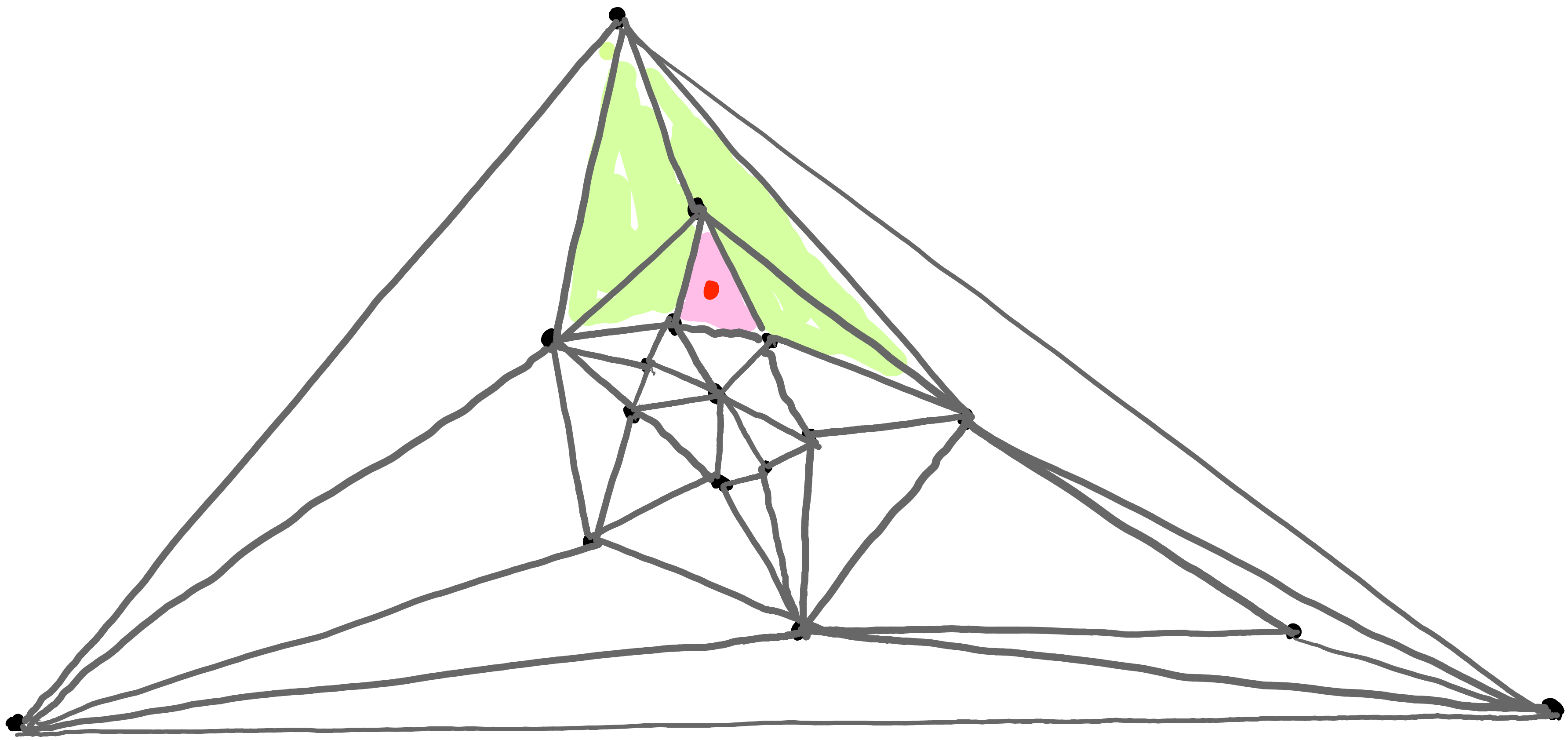
Level 3



Level 2

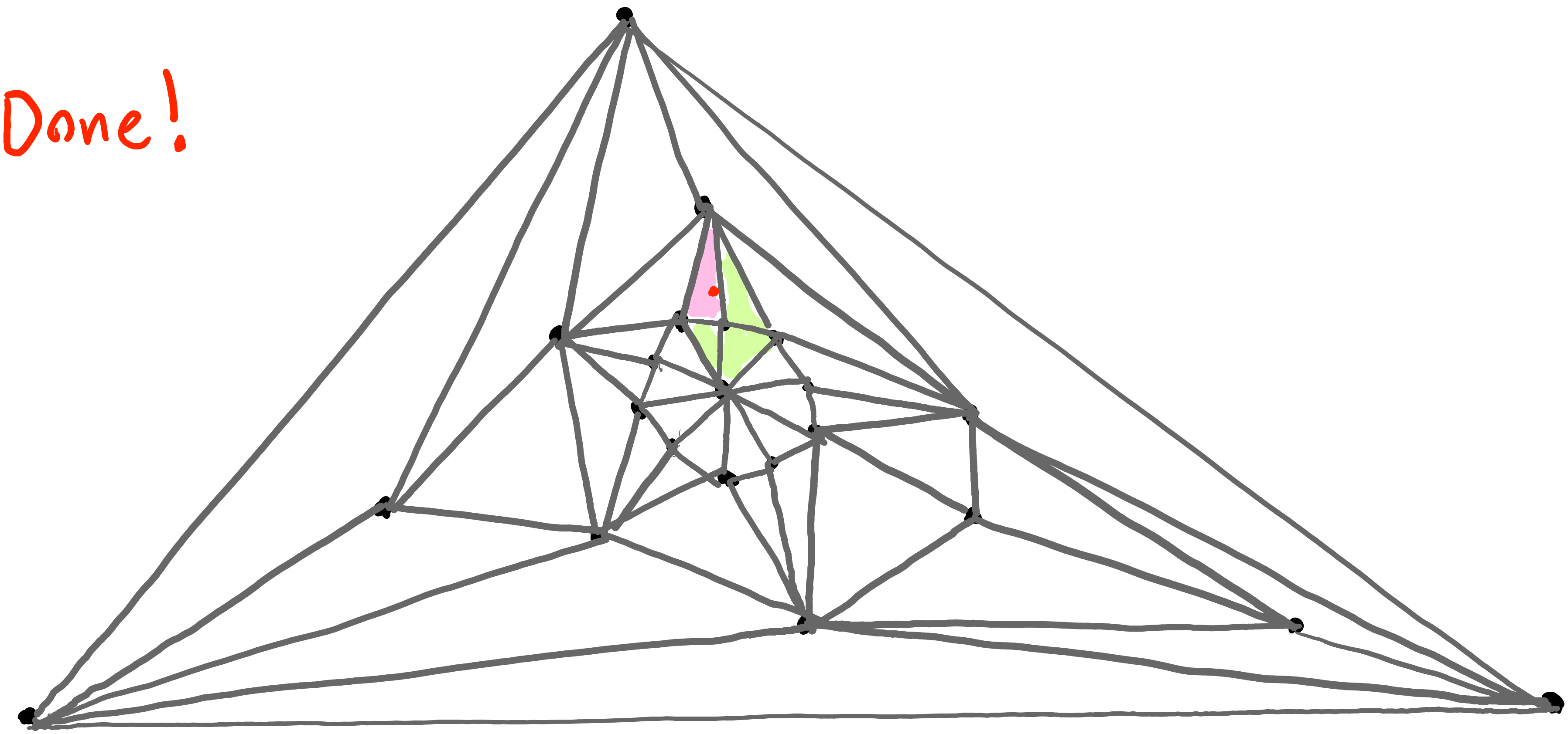


Level 1



Level 0

Done!



Summary

Summary

+ Construction time $O(n)$

Summary

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+ search $O(\log n)$

Summary

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− Constants terrible
− Requires triangulation
+ Can triangulate any simple polygon in $O(n)$ time [Chazelle]

Summary

+ Construction time $O(n)$
+ Search $O(\log n)$
+ Deterministic
+ Only method with this combination
- Constants terrible
- Requires triangulation
+ Can triangulate any simple polygon in $O(n)$ time [Chazelle]
- Chazelle's method is difficult

Summary

+ Construction time $O(n)$

+ Search $O(\log n)$

+ Deterministic

+ Only method with this combination

− Constants terrible

− Requires triangulation

+ Can triangulate any simple polygon in $O(n)$ time [Chazelle]

− Chazelle's method is difficult

− No obvious way to make dynamic